**Math 1031, Self-Evaluation Exercise 3: Solutions**
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**Name:** *Amy's Solutions*

**Discussion Section:** *NA*

**Discussion TA:** *NA*

This exercise is for your practise. There are six open-ended problems. Give yourself 20 minutes to complete the exercise, and see how you do.

1. Dan, Yun, and Ben walk into Starbucks. Each picks one of the following: a latte, an espresso, a capucino, a hot chocolate, a hot tea. How many possible outcomes are there?

   Think of this as three successive choices: Dan's choice, Yun's choice, and Ben's choice. Each person has 5 options, so there are $5 \cdot 5 \cdot 5 = 125$ possible outcomes.

2. If no number contains repeated digits, how many numbers greater than 3,000 can be formed by choosing from the digits 1,2,3,4?

   Think of this as four successive choices: choosing the first digit, choosing the second digit, choosing the third digit, and choosing the fourth digit.

   The first digit must be a 3 or a 4, so there are 2 possibilities for the first choice. The second digit can be any of the remaining digits, so there are 3 possibilities. Similarly, there are 2 possibilities for the third digit, and 1 possibility for the fourth digit. So there are $2 \cdot 3 \cdot 2 \cdot 1 = 12$ possible numbers that can be formed in this way.

3. A soccer coach chooses a Best Offensive Player, a Best Defensive Player, and a Most Improved Player from a team of 20 players. How many possible outcomes are there?

   There are 20 choices for the BOP, 19 choices for the BDP, and 18 choices for the MIP. So there are $20 \cdot 19 \cdot 18 = 6840$ possible outcomes.

   Notice that this is a *permutation* $\mathrm{P}(20, 3)$.

4. An art critic nominates ten promising young artists from a pool of 120 art students and gives four of them strong recommendations. How many possible ways can this be done?

   Think of this as two successive choices, choosing ten artists from 120, then choosing four from ten. There are $C(120, 10)$ ways to choose ten from 120, and $C(10, 4)$ ways of choosing four from ten, so there are $C(120, 10) \cdot C(10, 4) \approx 24$ quadrillion possibilities.

   Sorry about the inordinately long numbers in this problem! One quadrillion is 1,000,000,000,000,000.

5. Each distinct arrangement of the letters of the word MINNESOTA is written on a slip of paper and put in a hat. One slip is drawn at random from the hat. What is the probability that the slip constains an arrangement of the letters with the two Ns at the end?

   The sample space S is all the possible outcomes, i.e. all the possible arrangments of the letters of MINNESOTA. There are nine letters, but the two Ns are indistinguishable, so the number of arrangements is:

$$n(S) = \frac{9!}{2!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$$

   Let E be the event that the arrangement of letters has the two Ns at the end. We need to determine how many arrangements have the two Ns at the end. Well if we put the two Ns at the end we can put the remaining letters anywhere so

$$n(E) = P(7, 7) = 7!$$

   So the probability is

$$P(E) = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3} = \frac{2 \cdot 1}{9 \cdot 8} = \frac{1}{36}$$

6. Lucas and Angelica, along with ten other college students, are going to be mentors for twelve children at the YMCA. As they walk up to the YMCA, they see two kids playing outside. Assuming the kids *are* in the mentoring program, what is the probability that these two kids are the ones assigned to Lucas and Angelica?

   The sample space S is all possible ways of assigning mentors. Since there are twelve college students and twelve children, $n(S) = 12!$. The event E is the event that Lucas and Angelica are assigned to the two children playing outside. We can split this up into two mutually exclusive events, depending on which child is assigned to Lucas and which to Angelica.

   Suppose, for the sake of distinguishing between the two children, that one is a girl and the other is a boy. How many ways can we assign mentors such that Lucas has the girl and Angelica has the boy? Well, once those two are assigned there are ten mentors and ten children remaining, so there are 10! ways. Similarly, there are 10! ways to assign mentors such that Lucas has the boy and Angelica has the girl. So, all together, there are $2 \cdot 10!$ ways of assigning mentors such that Lucas and Angelica are assigned to the two children playing outside.

   So the probability that Lucas and Angelica are assigned to the two children playing outside is

$$P(E) = \frac{n(E)}{n(S)} = \frac{2 \cdot 10!}{12!} = \frac{2}{12 \cdot 11} = \frac{1}{66}$$