Math 321 Differential Equations Midterm Exam

Instructions. You will take this exam in class and have the opportunity to retake it outside of class. During the in-class time, writing instruments and a basic scientific calculator are the only resources you may use. During the outside-of-class time, you may also use the text and supplementary materials, class notes, Moodle resources, and CoCalc/SageMath. From the outside-of-class time, you may submit solutions to any number of exercises. These are due 5:00 PM, Thursday, October 17 in SC 117 or uploaded as a pdf in Moodle. If an outside-of-class solution is submitted for an exercise, then its grade will be averaged with the in-class grade. Each exercise is worth 10 points.

1. Write a differential equation model for the population of tribbles on the Starship Enterprise based on these assumptions: the growth rate of tribbles without harvesting is proportional to the square of their population, tribbles are harvested at a constant rate, and the initial population of tribbles in terms of biomass is 17 grams. State the meaning and units of each symbol used in your model.

2. Obtain the analytic solution to the differential equation $u'(t) = u(t)^3 \cos(t)$ with initial condition $u(0) = 3$.

3. Obtain the analytic solution to the differential equation $u'(t) = u(t) + 4e^{3t}$ with initial condition $u(0) = 7$.

4. The differential equation $tu'(t) = u(t) - t^2$ and initial condition $u(1) = 3$ have a solution of the form $u(t) = at^2 + bt + c$. Find it.

5. Consider the differential equation $u'(t) = t\sqrt{|u(t)|} + t^2$. For what initial conditions $u(t_0) = u_0$ does the Existence and Uniqueness Theorem assure the existence and uniqueness of solutions to the initial value problem? Explain.

6. Consider the differential equation $u'(t) = t\sqrt{|u(t)|} + t^2$. On a tu-plane, find where u' is zero, positive, and negative, sketch a possible solution curve for the initial condition $u(-4) = 1$, and sketch a possible solution curve for the initial condition $u(-4) = -10$.

7. Consider the differential equation $u'(t) = t\sqrt{|u(t)|}+t^2$. With the initial condition $u(0) = 9$ and a step size $h = 0.2$, calculate two steps of Euler's method.

8. For the differential equation $u' = u(u + 2)^2(u + 4)$, find all equilibrium points, determine their types, and sketch the phase line.

9. Obtain the general analytic solution to the differential equation $u''(t) + 2u'(t) + 5u(t) = 0$.

10. Obtain the analytic solution to the differential equation $u''(t) + 2u'(t) + 5u(t) = 10t + 9$ with initial conditions $u(0) = 0$ and $u'(0) = 0$.

11. Write a differential equation for a forced underdamped harmonic oscillator that is in resonance.

12. The amount $u(t)$ in moles per liter of a chemical t seconds after the start of a reaction is thought to be modeled by the equation $u(t) = u_0 e^{-kt}$. By taking the natural logarithm of both sides of this model, we obtain $\ln(u(t)) = \ln(u_0) - kt$. From the graph, estimate u_0 and k. Include their units.