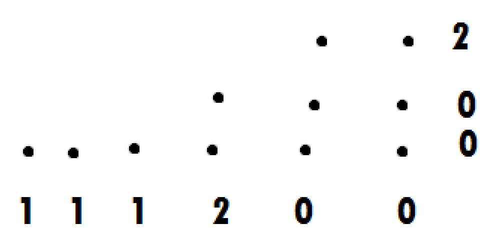
Garrett Ahlgrim

The problem that I chose to investigate was the variation on the game of Nim entitled "The Princess and the Roses". The game, complete with context, is as follows:

Princess Alice has two suitors: Laura and Renata. They alternate days in trying to gain her heart by bringing her roses. They pick up the roses from the same garden. Each day, the corresponding suitor will bring her one or two roses, but never two roses of the same colour. The suitor to pick the last rose from the garden will win Alice's heart. Assuming that initially there are 3 blue roses, 4 red roses, 5 yellow roses, and 6 white roses, who will win?

Now, my first instinct when it came to analyzing the game was to approach it as though it were a game of nim. Quickly, however, I realized that the rules that were set up prevented the winning strategy from behaving as though it were a game of Nim, and I found no way to wrap those two together. So I started analyzing game trees, starting with small games that only involved a couple of columns, or colors of flowers as the story would have it. No clear pattern really emerged to me.

Eventually, I just decided to stop doing small little games and just straight into the meat of what I was trying to analyze in the first place, rather than trying to work my way up to it. That' when I started with a simple row of six dots. After playing out the game in a game tree, I realized that I'd played this game before. It was the game of poison, where there is a column of beans, and each player can take either one or two beans, and the last player to take a bean loses. The only difference here was that the last player to take a dot wins. So I went out on limb and analyzed it as though it were a game of poison rather than nim. After having looked up the winning strategy and remembering that it simply boiled down to working in trinary, I attempted to expand the general principles of poison into other rows and pretend they were their own games of poison, so I would label games as you see below:



Sum of the numbers mod3: 1

Each of the numbers in the rows and columns represent how many dots are there modulo 3, as per poison.

It turned out that there were a couple of clear patterns. First of all, player one had the winning strategy in the handful of games that I plotted out when the sum of all the rows and columns was either 1 or 2. If the starting sum was 0 mod 3, then player two would win. As I began to analyze the behavior of the numbers as dots got removed, this pattern began to make sense. Removing one dot essentially means subtracting one from two of the row/column numbers. Now, if both of those numbers are either 1 or 2, then the total sum will drop by 2. In the event that you are subtracting from a 1 or a 2 from one row/column and a 0 in the other, then the net effect is to increase the modular sum by 1. If you are subtracting one from two zeros, then the sum increases by four, which reduces to increase by one. Thus, removing a dot only allows you to either increase the total sum by 1 or to decrease it by 2. Furthermore, if you take two dots, then in a similar fashion, you can either increase by 2 total or decrease by one total. It is not until taking a third dot off that you can get back to the original modular sum. This then shows that if the goal on any player's turn is to change the modular sum to 0, they can only do so if the current sum is 1 or 2. A player that has to remove dots from a game where the modular sum is already 0 won't be able to get to 0 again, and is thus unable to get back the control of the game from the other player.

Now, realizing that I haven't really proven that the winning strategy for the game is to change the modular sum to 0 on your turn until you win, I can actually do the next best thing: show that this is not in fact a variation on the game of poison. It *is* the game of poison. This may not be completely clear at first, but consider what happens when we take a dot from one of the upper rows and place it down in a new column. The net effect is that you are going to be subtracting one from a row and from a column when you take it off, but when you set it back down, you add one for the newly created column and add another for increasing the number of dots in the base row. Thus, when looking at the modular sum, the important number for determining your next move, it will go down by two and then come right back up by two, having a net effect of 0. Repeat the process until all of the dots are in one row, and suddenly the game becomes the game of poison, and the proof for the winning strategy holds. This can be done for any game with any number of columns and any number of dots in those columns. Therefore, the game of the Princess and the Roses is nothing more than a complicated façade for a very simple game.