

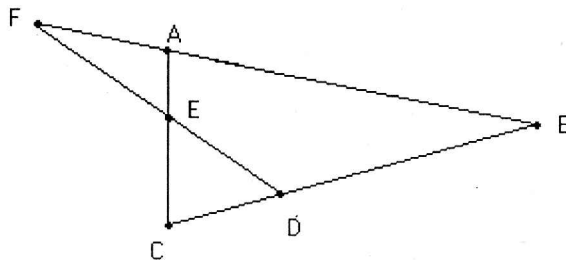
Menelaus Theorem

I chose to present the Menelaus Theorem, finding an interesting reference on the Internet about a letter from Albert Einstein to his friend Max Wertheimer. Apparently they were discussing the beauty of proofs, and Einstein used two proofs of the Menelaus Theorem, one ugly, and the other elegant, (in his opinion) to illustrate his point. I will discuss and analyze these proofs in this paper.

Einstein wrote, (of a proof) "...we are completely satisfied only if we feel of each intermediate concept that it has to do with the proposition to be proved."

The Menelaus Theorem states:

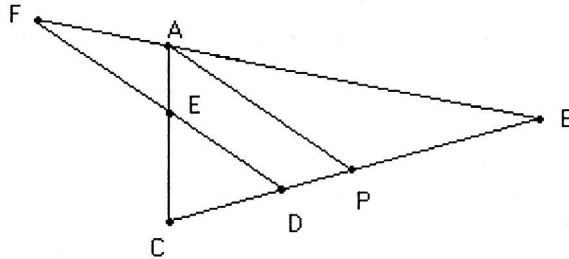
Let three points F, D, and E, lie respectively on the sides AB, BC, and AC of $\triangle ABC$. If the points are collinear, then $AF/BF * BD/CD * CE/AE = 1$.



As you can see, the theorem allows, and requires, that at least one of the points lie on the extension of the corresponding side, since a straight line cannot cross all three sides of a triangle. (On an interesting side note, in a 1945 Russian math competition, one boy did not see the fact that a straight line can't cross a three sides of a triangle as obvious. For that insight, he was awarded first prize, even

though he didn't solve a single problem.) This "If, then" statement requires a direct proof.

Proof 1.



Suppose ABC is a triangle. Suppose F, D, and E are points lying on sides AB, BC, and AC of the triangle ABC, respectively. Suppose F, D, and E are collinear. Consider AP, parallel to DE, with P lying on side BC of $\triangle ABC$. Since AP and DE are parallel, triangles ABP and BDF are similar, as are triangles ACP and CDE. Since $\triangle ABP$ and $\triangle BDF$ are similar, $AF/BF = PD/BD$. Since $\triangle ACP$ and $\triangle CDE$ are similar, $AE/CE = PD/CD$. Then $AF/BF * BD/PD = 1$ and $AE/CE * CD/PD = 1$. Also, $(AF/BF * BD/PD) * (CE/AE * PD/CD) = 1$. So, $AF/BF * (BD * PD / PD * CD) * CE/AE = 1$, and thus, $AF/BF * BD/CD * CE/AE = 1$.

How would this proof work if all 3 of D, E, F be an extension of the side

Einstein states of this proof, "Although the first proof is somewhat simpler, it is not satisfying. For it uses an auxiliary line which has nothing to do with the content of the proposition to be proved, and the proof favors, for no reason, the vertex A, although the proposition is symmetrical in relation to A, B, and C. The second proof, however, is symmetrical, and can be read off directly from the figure." Many times in this course, we have felt the dissatisfaction and discontent

