THE NAME OF THE GAME

1988 Summer REU Project

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We would like to give many thanks and much gratitude to Professor David Housman of Worcester Polytechnic Institute, the "driving force" behind this project. Without whose seemingly unbounded energy and dedication to the pursuit of perfection, the value of our three person coalition would have been zero.

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July 1988

Introduction

An n-person cooperative game is a pair (N, v) where N = {1, 2, ..., n} is a set of n players 1, 2, ..., n and where v is a real-valued characteristic function on 2^N, the set of all subsets of N. Let S, a subset of N, be a coalition of players, and let v(S) assign a value to the coalition S when the members of S work together. Define $v(\emptyset) = 0$. The game (N, v) is called a value game. A cost game is defined as c(S) = - v(S).

A game is superadditive if for all coalitions S and T where $S \cap T = \emptyset$, $v(S \cup T) \geq v(S) + v(T)$.

A game (N, v) is additive if it can be decomposed into two games (N_1, v_1) and (N_2, v_2) such that, for all coalitions $S_1 \subset N_1$ and $S_2 \subset N_2$, $v(S_1 \cup S_2) = v_1(S_1) + v_2(S_2)$.

A game is said to be monotonic if $v(S) \geq v(T)$ whenever T is contained in S.

A vector $x = (x_1, x_2, ..., x_n)$ with real components is an imputation for the game if $x_i \geq v(i)$ for all i contained in N (individual rationality), and

$$\sum_{i=1}^{n} x_i = v(N) \ (efficiency).$$

(See figure 1.)

An imputation x is coalitionally rational if, for all $S \subset N$,

$$\sum_{i \in S} x_i \geq v(S).$$

The core of a game is defined as the set of all imputations x such