COOPERATIVE GAMES ON WEIGHTED GRAPHS

by

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Abstract: The class of weighted graph cooperative games is described and it is shown that the Shapley value, tau value, and nucleolus give identical allocations on these games. A more general sufficient condition is stated for these three allocation methods to yield identical allocations.

Introduction

An n-person cooperative game is a pair (N, v) where $N = \{1, 2, ..., n\}$ is a set of players and where v is a real-valued function on the set of all subsets of N, where $v(\emptyset) = 0$. We interpret $v(S)$ as the value obtainable by the coalition S =when the members of S work together. A game is superadditive if for all coalitions S and T where $S \cap T = \emptyset$, $v(S \cup T) \geq v(S) + v(T)$. We will use S^c to denote the complement of S: N\S.

Several special classes of games, defined with respect to a weighted graph or network, have been considered in the literature. Bird [1], Megiddo [6], Granot and Huberman [3], Rosenthal [11], and others have studied minimum cost spanning tree games where the players correspond to the regular vertices of a weighted graph and $v(S)$ is the negative of the minimum weight tree that spans S and a special vertex. Kalai and Zemel [4], Dubey and Shapley[2], and others have studied network flow games where players control subsets of edges in a network and $v(S)$ is the maximum flow through the subnetwork induced by the edges controlled by players in S. Shapley and Shubik [14] and others have studied the assignment game where players are the vertices of a weighted bipartite graph and $v(S)$ is the maximum weight matching on the subgraph induced by S. Potters, Curiel and Tijs [10] have studied traveling salesman games where the players correspond to the regular vertices of a weighted graph and $v(S)$ is the negative of the minimum weight circuit covering S and the special vertex. Myerson [7], Owen [9] and Rosenthal [11] considered games in which cooperation is limited by a "communication" graph. In this paper, we study games in which the players are the vertices of a weighted graph and $v(S)$ is the sum of the weights on the edges of the subgraph induced by S.

A vector $\mathbf{x} = (x_1, x_2, ..., x_n)$ with real components is an imputation for the game if $x_i \geq v(i)$ for all i contained in N (individual rationality), and

$$\sum_{i=1}^{n} x_i = v(N) \quad (\textit{efficiency}).$$

An allocation method is a function from games to imputations. In this paper we consider three of