A Value for Zero-monotonic Partially Defined Games

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August 26, 1994

Introduction

A *cooperative game* is a pair (N,v) where N={1,2,...,n} and v is a real-valued function on the nonempty subsets of N. The elements of N are generally called *players*, and the subsets of N are called *coalitions*. We call v the *worth function*, and so v(S) is interpreted as the worth of the coalition S. In other words, v(S) is the amount that the players in S can jointly produce through cooperation.

The *zero-normalization* of a game (N,v) is the game (N,u) where $u(S) = v(S) - \sum_{i \in S} v(\{i\})$. It follows that a game is zero-normalized if v({i})=0; that is, the worth of each singleton coalition is equal to zero. A game (N,v) is *monotonic* if $v(S) \leq v(T)$ for all $S \subseteq T \subseteq N$. Moreover, a game is said to be *zero-monotonic* if its zero-normalization is monotonic. Monotonicity is a favorable property as it assures that the addition of a player will not lessen the worth of a coalition. We shall assume zero-normalization and zero-monotonicity throughout this report.

One goal of cooperative game theory is to find a fair method of distributing joint savings or costs among the players involved in a venture. An *allocation method*, or *value*, is a function that assigns to each game (N,v) an *allocation* $x = (x_1, x_2, \ldots, x_n)$ where x_i is the fair share, or *payoff*, of the total benefit v(N) that player i receives for its cooperation in the group. One commonly used allocation method is the Shapley value, which is given by the formula: