

The Island of Truthtellers and Liars

Conjecture and Proof
Research Paper
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You have found yourself upon The Island of Truthtellers and Liars.

Assumptions:

1. All questions must be "yes and no" questions and have a single right answer.
2. Islanders know the answer to every "yes or no" question.
3. All islanders are either Truthtellers or Liars.
4. Everyone from the tribe of Truthtellers always tell the truth
5. Everyone from the tribe of Liars always lies.

Definition 1: Subjective Questions are questions whose correct answer changes depending on the person being asked. "Is your name John?" "Are you from the tribe of Truthtellers?" or "Do you like lasagna?" are subjective questions.

Definition 2: Objective Questions are questions that are not subjective (a.k.a. the correct answer does not change with the person you are asking. Questions such as "Is the sky blue?" or "Is Cityopolis to the north?" are objective questions.)

Theorem 3: All questions are either subjective or objective.

Proof: By definition, all questions that are not subjective are necessarily objective. Therefore, all questions are either subjective or objective.

Theorem 4: In answering an objective question a Truthteller and a Liar will always give different answers.

Proof: Suppose you ask any question to two islanders, knowing that one of them is a Truthteller and one is a Liar. Then, because the Truthteller always tells the truth, she will give the correct answer. The Liar, who always lies will give a false answer. The false answer cannot be the same as the correct answer, or it would be the correct answer, and the Liar would have told the truth. Therefore, the Truthteller and the Liar would give different answers.

Theorem 5: If two islander answers an objective question differently then they are from different tribes.

Proof: Suppose two Truthtellers are asked a question. Since they will both give the correct answer, and there can only be one answer, the answers will be the same. They will either both say "yes" or both say "no." If the two islanders are Liars then the first liar will answer the question with the wrong answer. If the correct answer is "yes" that islander would say, "no" and if the correct answer is "no" then she will say "yes." The other liar would do the same. Therefore, both liars would say the same thing either way. Since we know that in answering an objective questions members from different tribes

