FAIR ALLOCATION METHOD FOR STEINER TREE NETWORKS

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Communications networks are costly to build and so the users of the network must contribute to the development and maintenance of the network. It would be good to allocate costs so that no group of users is asked to contribute more than they did if they were to build a network to meet their own needs. Such an allocation is called group rational. Unfortunately, group rational allocations are not always possible. This paper provides a computationally efficient algorithm for finding a group rational allocation in a special class of network configurations.

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Allocation Method for Steiner Tree Networks

A cost game is defined as a set N = {1, 2, … n} of players and cost c(S) associated with each subset S of players. Example:
N = {1, 2}
c(1) = 2 → cost of player 1 alone
c(2) = 3 → cost of player 2 alone
c(12) = 4 → cost if player 1 and 2 collaborate

The problem that arises from this cost game is how to allocate the total cost, when all players collaborate, to each player. Let us denote the allocation for a game as $x = (x_1, x_2, \ldots, x_n)$ where x_i is the allocation for player i.

Core

An allocation is said to be in the core if the sum of the allocation of players in any subset S of N is ~~less~~ no greater than the cost for that subset.

$$\sum_{s \in S} x_s \leq c(S), \quad \forall\, S \subset N$$

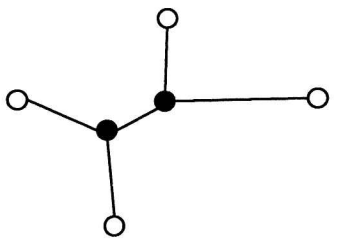
There are 2 common methods that are used to decide the allocation:

1. Shapley value which takes the average of the marginal contribution of each player over all possible orders. Marginal contribution for a player i is the ~~amount~~ cost that a player i adds to a coalition when player i joins the coalition ($c(S \cup \{i\}) - c(S)$ for $i \notin S$. The Shapley value is not always in the core.
2. Nucleolus which lexicographically minimize the maximum coalitional complaints. Nucleolus is always in the core provided the core is not empty.

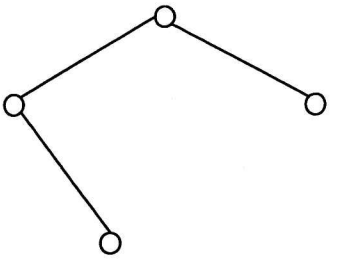
provide references

Steiner tree

Given a set of points. Steiner tree is a tree that connects all points in S with possibility of having some additional points, called Steiner points, not in S. Example:



Stiner tree



Spanning tree

Explain that a Steiner point is of degree ≥ 3.

Rectilinear Steiner tree is a Steiner tree that consist of only horizontal and vertical lines as the connector.