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THE NUCLEOLUS AND SHAPLEY VALUE FOR COOPERATIVE MATCHING GAMES ON WEIGHTED GRAPHS

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Abstract: In any cooperative game we are often concerned with fairly allocating the savings to the players in the game. This paper introduces a specific cooperative game, called the matching game, and a procedure for finding the nucleolus and Shapley value (two fair allocations) for different classes of matching games.

Introduction: An n-person cooperative game is a pair (N,w), where $N = \{1,2,3,...,n\}$ is the set of players and $w(S)$ is a real-valued function from the subsets of N to the real numbers which satisfies $w(\emptyset)=0$. The subsets of N can be thought of as coalitions of players in N, and w can be interpreted as the worth function because $w(S)$ represents the relative worth of the coalition S. A cooperative game is called *superadditive* if $w(S \cup T) \geq w(S) + w(T)$ for all $S,T \in N$ satisfying $S \cap T = \emptyset$. More plainly, superadditivity ensures that when two or more disjoint players or coalitions decide to cooperate their worth will be greater than or equal to the sum of their individual worths.

Many cooperative games can be defined with respect to graphs. A *graph* is a finite nonempty set of objects, called *vertices*, together with a possibly empty set of unordered pairs of distinct vertices called *edges*. In a graph-restricted game