Values on Partition Function Form Games

Stephanie Merki
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Partition function form games were first introduced in Lucas and Thrall in 1963 as a generalized form of characteristic function form games. Both R.B. Myerson and E.M. Bolger have defined values on partition function form games. In this paper an extension of the Shapley value is sought which will satisfy linearity, efficiency, symmetry and dummy. Different axioms are then incorporated to place bounds on the various remaining parameters.

The following background information, along with 12 axioms and 7 definitions, is key to the paper.

<u>Background Information</u>:

$N = \{1,2,\ldots,n\}$ is the set of players in a n-person game.

$CL = \{S \mid S \subseteq N, S \neq \emptyset\}$ is the set of coalitions of N.

PT = set of partitions of N: $\{S^1,\ldots,S^m\} \in PT$ iff

$S^1 \cup \ldots \cup S^m = N$, $\forall j\ S^j \neq \emptyset, \forall k\ S^k \cap S^j = \emptyset$ if $k \neq j$.

ECL = set of embedded coalitions: $\{(S,P) \mid S \in P \in PT\}$.

A game in partition function form is any $W \in R^{ECL}$.

$W(S;P)$ is the amount S would receive if partition P formed.

$\Phi(W)$ is a payoff vector for the game W.

$\Phi_i(W)$ is a payoff or allocation to player i on game W.

A game is w-superadditive if

$W(S;P)+W(T;P) \leq W(S \cup T; P-\{S,T\} \cup \{S \cup T\}) \forall (S;P),(T;P) \in ECL$.

A game is w-coalition monotonic if

$W(S;P) \leq W(T;\{Q-T : Q \in P\} \cup \{T\}), S \subseteq T$.

A game is w-partition monotonic if

$W(S;P) \leq W(S;Q)$ whenever Q is a refinement of P, i.e.,

$R \in Q \Rightarrow \exists S \in P$ such that $R \subseteq S$.

$(T;Q) > (S;P)$ if $S \subseteq T$ and $Q-\{T\}$ is a refinement of $P-\{S\}$.

$\{R-T : R \in P-\{S\}\}$