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Math 413

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Modified Collatz Conjecture

For this project, I will modify the well known problem called the Collatz conjecture. For the Collatz conjecture, you can start with any positive real integer, n. If it is odd the function will return 3n +1. If the number is even, the function will return n/2. For any initial n value, if the out puts continue to be put back in the function, you will eventually get the output of one. Once you get to the number one, it will create a loop of the numbers one, four, two, and then one again. This conjecture has never been proven, but people have shown that this is true for all numbers less than or equal to 2^{40}.[1] Many upper level mathematicians have been working on this problem for years and I don't believe I would be able to prove it in just one semester. For this paper I will rather be analyzing how the results vary for different values of natural numbers b. I will call this function f(n) sub b where if n is odd, the output is 3n + b and if n is even, the output is n/2. I will only be looking at values of n that are natural numbers. When b = 1, we have the original Collatz conjecture, so I will be looking at when b is a natural number greater than one.

Let's start by looking at when b is an even number. When the number we are given to start with is even, the first step will be to divide it by 2. We will continue to do this until we have an odd number. We know that any even natural number divided by two will give us a natural number. If it gives us an even number than we will just divide by two again. Once we have an odd number, we then multiply that odd number by three. This will give us a greater odd number

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[1]Stefan Andrei, and Cristian Masalagiu. 1998. "About the Collatz Conjecture." *Acta Informatica* 35 (2): 167. doi:10.1007/s002360050117.

because an odd number times an odd number will always result in an odd number. Then we add two to it. Any time you add two to an odd number, the result will be a greater odd number. Since all odd numbers will result in getting a bigger odd number, this will continue forever. Thus, as you continue to plug the outputs back into the function, the outputs will approach infinity.

For any odd value of b, there exists an infinite loop containing b as the smallest element in that loop. Because it is odd, f sub b will get multiplied by 3 and then have b added to it. This is equal to 4 times b. 4b is even and when it gets plugged into f sub b it will be divided by two and result in 2b. 2b is also even and will be divided by two when it gets plugged back in f sub b. This will result in b, an odd number. If we continue to plug the result into f sub b, we will get back 4b again and the cycle would continue. Thus, for any odd value b, there exists an infinite loop where b is the lowest value in it.

Any multiple of b that gets plugged in f sub b, when b is odd, will eventually get to an output that is b. A multiple of b can be written as kb where k is a natural number. We will plug kb into f sub b until we get an odd number, this number would be kb if kb is odd. We will refer to the odd number that we get as xb, were x is a natural number. We know that x is odd because if x were even, then xb would be even since an even number times an odd number has an even product. When we plug xb into f sub b, we get 3(xb) +b. This can be written as (3x +1)b. (3x +1) is the output that you would get from plugging the odd number x into the original Collatz Conjecture one time. 3 times an odd number is a bigger odd number and an odd number plus one is an even number. Thus, when we plug (3x +1)b into f sub b, we will get $\frac{(3x+1)b}{2}$. That is also equal to $\frac{(3x+1)}{2}$b . This value could be even or odd, depending on the values of the variables. If it is an even number, keep plugging it into f sub b until the output is odd. This value can be represented as $\frac{(3x+1)}{2^n}$b, where n is a natural number. When we plug $\frac{(3x+1)}{2^n}$b into f sub b, we get

$3(\frac{(3\square+1)}{2\ \square}b)+b$. This can also be written as $(3(\frac{(3\square+1)}{2\ \square})+1)b$. Thus, when ever we plug an even out put into f sub b, we only divide the number multiplying b by two. When the output is odd, we know that it is a multiple of b, because the original value was and so are all of the outputs. So we can factor out the b from the input and b. Then when we have an odd number input, we multiply that value by three, add one, and then multiply it all by b. In every output, the value being multiplied by b is the value that it would be after the same number of times being plugged into the Collatz Conjecture. Assuming the Collatz Conjecture is true, the number multiplying b will converge to one as it continues to get plugged into f sub b. Thus, the output will eventually be b. So any multiple of b that gets plugged into f sub b, when b is odd, will converge to a loop where b is the lowest value in it.

Now we will look at when b is equal to 3. No matter whether we start with an even or an odd number, we know from above that eventually we will reach an odd number because for even numbers you just divide by two until you reach an odd natural number. When you have an odd number, n, you multiply it by 3 and add 3. This is equal to 3(n + 1). Because n is an odd natural number, we know that n+1 is an even natural number. We will represent this even natural number as 2k where k is a natural number. So we have 3(2k). This is equal to 6k. We will know this number is even, so the next time it goes through the function, it will be divided by 2. Then we have 3k. Because k is a natural number, the smallest value it can be is 1. Thus the smallest value that is in an infinite loop is 3 because three can not be divided by 2.

I plugged in the initial values of 1 through 1,000 into f sub b for multiple different b values and made some conjectures from my findings.

- I conjecture that for all initial n values, f sub 3 will lead to a loop where the lowest value in the loop is 3.

- I conjecture that for all initial n values that are not divisible by 7, that f sub 7 will lead to a loop where the lowest value in the loop is 5.
- I conjecture that for all initial n values, f sub 9 will lead to a loop where the lowest value in the loop is 9.
- I conjecture that for all initial n values that are not divisible by 19, f sub 19 will lead to a loop where the lowest value in the loop is 19.
- I conjecture that for all initial n values that are not divisible by 7, f sub 21 will lead to a loop where the lowest value is 15.
- I conjecture that for all n divisible by 7, f sub 21 will lead to a loop where the lowest value is 21.

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Works Cited

Andrei, Stefan, and Cristian Masalagiu. 1998. "About the Collatz Conjecture." *Acta Informatica* 35 (2): 167. doi:10.1007/s002360050117.