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Consistency: An Axiomatic Classification of Allocation Methods on TU Characteristic Function Form Games

Abstract: For several well-known functional allocation methods, there exists an axiomatic description in which the important property is consistency. Consistency stipulates that given a subset of players T, an allocation method returns the same allocation for each player in the reduced game on T as it does on the large game. Thus the study of consistency properties is simply a study of reduced games. In general, a specific reduced game uniquely defines an allocation method, given a few additional weak assumptions. Thus it would seem that a classification of reduced games can provide a classification of allocation methods. The literature so far has only provided special cases of reduced games. In this paper a general reduced game form is postulated. A subclass of the postulated reduced games are shown to classify a class of linear allocation methods (which includes the Shapley Value). The subclass of reduced games is then shown to be a convex combination of simple reduced games whose associated allocation methods span the derived class. An axiomatic characterization of the class of weighted prenucleoli (including the prenucleolus and the per capita prenucleolus) is also given using another subclass of the postulated reduced games.

Definitions:

Denote a game in characteristic function form (or simply game) by the ordered pair (N,v) where N=(1,...,n) denotes the set of players ($|N|=n$) and v denotes the real valued function $v:2^N \longrightarrow R$. Define a subgame to be the game on a subset of players, ie. (S,v) for $S\in N$. An allocation or pre-imputation is a vector $0\neq x=(x_1,\ldots,x_n)\in R^n$ satisfying $\sum_{i\in N} x_i=v(N)$ (Efficiency or Pareto Optimality).
An allocation method θ, or simply a solution, will denote a relation identifying a game (N,v) with a subset of the set of allocations for that game. Denote the image of θ by $\theta(N,v)$. There are a number of simple properties that can restrict the set of solutions under consideration.

An allocation method θ is defined at the game (N,v) if (N,v) is part of the domain of θ.

(ETP) An allocation method satisfies the Equal Treatment Property if $\theta(N,v)_i=\theta(N,v)_j$ when $i,j\in N, i\neq j$, satisfy $\theta(S\cup\{i\})=\theta(S\cup\{j\})$ for all $S\subset N\setminus\{i,j\}$.

(SYM) Let $p(N)=[p(1),\ldots,p(n)]$ denote a permutation of the players in N. Define the value function pv by $pv[p(S)]=v(S)$ for all $S\subseteq N$. An allocation method satisfies the Symmetry Property if $p(i)(N,pv)= i(N,v)$ for all permutations p.

(COV) A solution is defined to be proportionate if for $a\geq 0$ and worth av defined by $(av)(S)=av(S)$ for all $S\subset N$, it