

Cooperative Games in  
Partition Function Form

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## Partition Function Form (PFF) Games

### Introduction

Games in partition function form differ from those in coalition function form in that the worth assigned to each coalition depends on how the rest of the players collude. Although little is known about the "reasonableness" of values for coalition function form games, natural extensions of the Shapley value and the nucleolus are considered for PFF games. In my research various concepts of reasonable outcomes are defined for these games based on those defined by Milnor for coalition function form games. Also an allocation method analogous to the nucleolus is sought for PFF games.

### Background Information

We begin with a set of players  $N = \{1, 2, \dots, n\}$  and consider all coalitions  $S \subseteq N$ . A game  $W$  in partition function form assigns a real value, or worth, to each subgroup  $S$  depending on the partition to which  $S$  may belong. To define  $W$  formally, we first introduce the following sets:

- The set of possible coalitions,

$$CL = \{S \subseteq N : S \neq \emptyset\}$$

- The set of possible partitions,

$$PT = \{P : P \text{ is a partition of } N\}$$

$\{S_1, S_2, \dots, S_k\}$  is a partition of  $N$  iff:

- $\emptyset \neq S_i \subseteq N$ , for each  $i = 1, \dots, k$ .
  - $\forall i \in N, \exists k$  such that  $i \in S_k$ .
  - $S_i \cap S_j = \emptyset \quad \forall i \neq j$ .
- The set of embedded coalitions,

$$ECL = \{(S; P) : S \in P \in PT\}$$

