

The Shapley Value and Partially Defined Games

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Introduction

Many mathematicians are realizing the usefulness of cooperative game theory in the process of allocating costs or benefits among the participants of joint endeavors. Realizing that in reality, the determination of all coalitional worths may be prohibitively expensive or impractical, this paper is aimed at providing possible allocation methods for games which have unknown coalitional worths.

Definitions

In this paper, all games in which some coalitional worths are not known are referred to as partially defined games, or PDG's. Each game consists of a set of players, N , with $N = \{1, 2, \dots, n\}$. Let M be a subset of N such that $1, n \in M$. A partially defined game with respect to M , or an M -game, is a real valued function ω on $\{S \subseteq N: |S| \in M\}$. The real number $\omega(S)$ is often called the worth of the coalition S . By defining M so that it always contains 1 and n , we insure that the worth of the singleton and grand coalitions are known, in addition to the worths of any other coalition whose size is in the set M .

An allocation for the M -game is a vector of payoffs $x \in \mathbb{R}^n$. An allocation method is a function from a class of games to allocations \mathbb{R}^n which attempts to fairly distribute the costs or benefits of the joint venture. The

