Intergenerational Games

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In certain cultures, decisions are made based on their effects on future generations. Rather than seeking ephemeral gains for themselves, policy makers at both the government and family levels project into the future and speculate as to how their decisions will effect their progeny. An extreme case is that of the Iroquois indians who ask "how will this decision effect our ancestors seven generations from now?" when making tribal decisions. Thoughts of these situations as well as realizations of recent global environmental trends has led me to consider a new class of cooperative games, games in which one player, the present decision maker, controls the game and the fate of the other players who are subjected to the abuses as well as the sacrifices of the dominant player.

My research involves what I will refer to as intergenerational games; that is, the players are the decision makers of each generation. I will analyze the effects of projecting into the future and considering the welfare of following generations in the decisions made by the present generation. My analysis will begin with the case of allocating a fixed, non-renewable resource with consideration for zero, one or two generations into the future. After presenting a specific instance, I will then generalize these results and prove that the Shapley value allocates an unattainable amount to the initial generation except under special circumstances. The next step in this research will be to analyze the case of renewable resources, the notion of overlapping generations, and competition within each generation.

The following assumptions will hold for the specific example and the general case which follows.

1. There exists a fixed amount, R_0 units, of some desirable resource, and a population of size N which consumes this resource.
2. Now or in the near future, scarcity of this resource will be a problem.
3. There is a decreasing per capita utility of using the resource.

4. There is an increasing marginal cost of using the resource.
5. Both the population and the resource have a zero growth rate. (this will be relaxed later on)

Define $U(r) = k_1(r/N)^{1/a}$: The utility for using r units of R_0

$C(r) = k_2(r/R_0)^b$: The cost of using r units of R_0

$P(r) = U(r) - C(r)$: The benefit of using r units

Taking a derivative of P(r) and setting it equal to zero yields the following result:

Formula 1: $$r^* = \left(\frac{k_1 \; R_0^{\,b}}{k_2 \; a \; b \; N^{1/a}} \right)^{a/(ab-1)}$$

The second derivative test indicates that r_* is a local maximum. Now I assign the value of the coalition g_i to be $P(r_i^*)$.

Example 1:

Let a=b=2 $R_0 = 1000$; $N = 50$; $k_1 = 1$; $k_2 = 2$;

Formula 1 gives $r_{1*} = 679$.
Hence $v(g_1) = 3.685 - 0.922 = 2.763$
For g_2 $R_1 = 321$, hence formula 1 gives $r_{2*} = 149$
thus, $v(g_2) = 1.726 - 0.431 = 1.295$