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FINDING A VALUE ON PARTIALLY DEFINED GAMES

Introduction

A cooperative game is a pair (N, v) where N = {1, 2, ..., n} and v is a real-valued function on the nonempty subsets of N. Elements of N are often called players while subsets of N are called coalitions. The grand coalition, denoted as N, is composed when all of the players cooperate together. The function v is called the worth function, and v(S) is interpreted as the worth of the coalition S. In other words, v(S) is the amount that individuals in S can jointly obtain if they cooperate as a group.

A game (N, v) is said to be superadditive if $v(S \cup T) \geq v(S) + v(T)$ for all disjoint coalitions $S, T \subseteq N$. The game (N, v) is monotonic if $v(S) \leq v(T)$ for all coalitions $S \subseteq T \subseteq N$. The game (N, v) is said to be 0-normalized if v(i) = 0 for all $i \in N$. The 0-normalization of a game (N, v) is the game (N, u) where

$$u(S) = v(S) - \sum_{i \in S} v(i) .$$

A game (N, v) is 0-monotonic if its 0-normalization is monotonic.

In an effort to find some standard of fairness or a predictor of bargaining solutions, many researchers of game theory have concentrated on finding the "best" way for individuals in a game to form coalitions and eventually maximize their savings. Once the savings have been made they must be allocated to the participating players. An allocation method is a function that assigns an allocation to each cooperative game in some class.

For every cooperative game there are $2^N - 1$ possible coalitions. Normally the worth of every coalition is known. However, this may not always be the case. For instance, a firm might have certain time or monetary constraints that prevent it from knowing the worth of all coalitions. The utility for a game of this type occurs when it is impractical to determine every coalitional worth, v(S). During the summer of 1990, while involved in a Research Experience for Undergraduates program at Drew University, David Letscher developed and began to research what might happen in such cases. He named this topic partially defined games.

Necessary definitions

A partially defined game, or PDG, is a triple (N, Z, v) where N = {1, 2, ..., n} is the set of players, Z is a collection of nonempty subsets of N, and v is a real-valued function on Z. We will often use v to denote a partially defined game when N and Z are clear from context.

Definition - Let J be a subset of the player set N such that 1, $n \in J$. A J-game is a partially defined game where $Z = \{S \subseteq N \mid |S| \in J\}$.

We will denote a J-game by (N, J, v). Z is the set of coalitions whose worths are known, while J is the set of sizes, or cardinalities, of coalitions whose worths are known. For example, in a 4 - player partially defined game where the worths of the grand coalition, the triples, and the singletons are known while the worths of the pairs are not, J = {1, 3, 4}. For this same game, Z = {A, B, C, D, ABC, ABD, ACD, BCD, ABCD}. In our research,