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Consistency: An Axiomatic Classification of Allocation Methods on TU Characteristic Function Form Games

Abstract: For several well-known functional allocation methods, there exists an axiomatic description in which the important property is consistency. Consistency stipulates that given a subset of players T, an allocation method returns the same allocation for each player in the reduced game on T as it does on the large game. Thus the study of consistency properties is simply a study of reduced games. In general, a specific reduced game uniquely defines an allocation method, given a few additional weak assumptions. Thus it would seem that a classification of reduced games can provide a classification of allocation methods. The literature so far has only provided special cases of reduced games. In this paper a general reduced game form is postulated. A subclass of the postulated reduced games are shown to classify a class of linear allocation methods (which includes the Shapley Value). The subclass of reduced games is then shown to be a convex combination of simple reduced games whose associated allocation methods span the derived class. An axiomatic characterization of the class of weighted prenucleoli (including the prenucleolus and the per capita prenucleolus) is also given using another subclass of the postulated reduced games.

Definitions:

Denote a game in characteristic function form (or simply game) by the ordered pair (N,v) where $N=(1,\ldots,n)$ denotes the set of players ($|N|=n$) and v denotes the real valued function $v: 2^N \longrightarrow R$. Define a subgame to be the game on a subset of players, ie. (S,v) for $S \in N$. An allocation or pre-imputation is a vector $0 \neq x=(x_1,\ldots,x_n) \in R^n$ satisfying $\sum_{i\in N} x_i = v(N)$ (Efficiency or Pareto Optimality).
An allocation method θ, or simply a solution, will denote a relation identifying a game (N,v) with a subset of the set of allocations for that game. Denote the image of θ by $\theta(N,v)$. There are a number of simple properties that can restrict the set of solutions under consideration.

An allocation method θ is defined at the game (N,v) if (N,v) is part of the domain of θ.

(ETP) An allocation method satisfies the Equal Treatment Property if $\theta(N,v)_i=\theta(N,v)_j$ when $i,j \in N, i \neq j$, satisfy $\theta(S \cup \{i\})=\theta(S \cup \{j\})$ for all $S \subset N \setminus \{i,j\}$.

(SYM) Let $p(N)=[p(1),\ldots,p(n)]$ denote a permutation of the players in N. Define the value function pv by $pv[p(S)]=v(S)$ for all $S \subset N$. An allocation method satisfies the Symmetry Property if $p(i)(N,pv)= i(N,v)$ for all permutations p.

(COV) A solution is defined to be proportionate if for $a \geq 0$ and worth av defined by $(av)(S)=av(S)$ for all $S \subset N$, it follows that $\theta(N,av)=a\ (N,v)$. Given $b_i \in R$ and $i \in N$, define a worth function u by $u(S)=v(S)+b_i$ for all $i \in S \subset N$ and $u(S)=v(S)$ for all $i \notin S \subset N$. A solution concept is defined to be value separable if $\theta_i(N,u)=\theta_i(N,v)+b_i$ and $\theta_j(N,u)=\theta_j(N,v)$ for $j \neq i$. An allocation method which is both proportionate and value separable satisfies the Covariance Property (or the Relative Invariance under Strategic Equivalence property), ie. for all a=0 and b Rn, $\theta(N,u)=a\theta(N,v)+b$ where u is defined by $u(S)=av(S)+\sum_{i \in S} b_i$ for all $S \subset N$.

Remark: Note that an allocation method satisfying several combinations of the above properties is already completely defined on two player games. We have the following well-known result:

Lemma 1: A solution on two player games $\theta(\{i,j\},v)$ is uniquely defined to be $\theta_i(\{i,j\},v)=v(\{i\})+1/2[v(\{i,j\}-v(\{i\}-v(\{j\})]$ if θ satisfies any of the conditions I,II,III or IV listed below;

I satisfies SYM and COV
II satisfies ETP and COV
III satisfies ETP and is value separable
IV satisfies SYM and is value separable

The proof follows easily from the definitions. A solution which evinces the above solution on two-player games will be referred to as standard on two person games. (See Driessen, (1990) and Hart & Mas-Colell, (1987))

In general, allocation methods fall into two categories. One class of solution concepts yield a set of allocations for each game. We will denote these solution concepts as Set-theoretic solutions. This paper is not concerned with this category of solution concepts. Allternatively, other solution concepts yield a unique vector for each game. We will denote these solution concepts as functional solutions.

Example 1 Equal allocation of joint value

The equal allocation of joint value (EAJV) first allocates each individual their individual worth v(i), and the amount remaining is divided equally among the members of N.

$$EAJV_i=v(i)+1/n[v(N)-\sum_{j \in N} v(j)]$$

Example 2: Equal allocation of nonseparable value

The equal allocation of nonseperable value (EANV) first allocates to each individual their separable value, ie. their marginal worth to the grand coalition, and then the amount remaining is divided equally among the members of N.

$$EANV_i = s_i + 1/n[v(N) - \sum_{j \in N} s_j] \text{ where } s_j = v(N) - v(N-\{j\})$$

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Example 3: The Shapley Value

The Shapley value for individual i is the average of the marginal values individual i brings to the set of players over all possible orderings. (see Shapley, 1953)

$$SH_i = \sum_{S \in N} \frac{(|S|-1)!(n-|S|)!}{n!} [v(S \cup \{i\}) - v(S)]$$

Example 4: The weighted nucleolus and weighted pre-nucleolus

Given an allocation x, define the excess of a coalition S (in (N,v)) by e(S,x,v):=|S|[v(S)-x(S)] where 0=w Rn and w|S| is the |S|th component; the excess is a measure of the power of the complaint that S can raise about x. Let e(x,v) be the vector of excesses e(S,x,v) S=0,N, ordered from highest to lowest. The weighed prenucleolus is the allocation which minimizes e(x,v) lexicographically. If wi=1 for all i, then is the prenucleolus. If wi=1/i then is the per capita prenucleolus. The weighted nucleolus minimizes e(x,v) lexicographically on the set of imputations. (see Schmeidler, 1969, and Grotte, 1970)

Introduction and Further Definitions:

The description of a game in essence consists of listing all the coalitional values. Thus it is quite natural to suppose that the description of an allocation method would be encapsulated in the action of the solution on all reduced games; ie. all the information of a game allocation is contained in the possible reduced games. Consistency requires that the solution returns the same allocation coordinate on a reduced game as on the full game. Thus, the definition of the reduced game determines the particular functional allocation method which is consistent on that reduced game. Adding sufficient basic properties to guarantee uniqueness then axiomatically characterizes the solution. The elemental nature of the consistency approach allows the mathematician to begin to compare different allocation methods quantitatively. There have been several axiomatic characterizations of allocation methods involving consistency in the literature. Sobolev (1975B) characterized the prenucleolus using the consistency property developed by Davis and Maschler (1965). This consistency property also plays a role in Peleg's characterization of the prekernel and the core. (Peleg, (1985)) The Shapley value can be

characterized by three different consistency properties (so far). (See Dreissen (1991)). The most useful for the purposes of this paper is the consistency property developed by Sobolev (1975A). More recently, Potts (1991) has shown that the egalitarian value can be characterized by both Davis & Maschler and Hart & Mas-Colell consistency (See Potts (1991)).
Formally,

Given k, $1<k<n$, a solution θ is k-player consistent with respect to the reduced game X (in other words, θ is k-player X-consistent) on a class G of games if

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$$\theta(N,v)=\theta(T,vT,\theta)$$

for all $T\subset N$ s.t. $|T|=k$, and games $(N,v)\in G$, where vT,θ is the X reduced game of (N,v) on T with respect to θ. A solution is X-consistent if it is k-player X-consistent for all k, $1<k<n$.

Given $\theta(N,v)=x$, the Davis & Maschler reduced game (T,vT,x) is defined as follows

$$vT,x(\emptyset)=0$$

$$vT,x(T)=x(T)$$

$$vT,x(S)=\max\{v(S\cup R)-x(R):R\subset N-T\} \text{ for } \emptyset\neq S\subset T.$$

Given $(N,v)=x$, the Hart & Mas-Colell reduced game (T,vT,x) is defined as follows

$$vT,x(S)=v(S\cup N\backslash T)-\sum_{i\in N\backslash T} i(S\cup N\backslash T,v) \text{ for all } S\subseteq T.$$

Results:

In order to motivate the introduction of the general reduced game form, results pertaining to two specific solution classes will be discussed initially. The first class of solutions is the one originally delineated by Maltenfort (1990) which consists of (in essence) symmetric linear allocation methods. The second class of solutions is the class of weighted prenucleoli. A general form including the special cases will then be introduced.

<u>A Class of Linear Allocation Methods</u>

Consider the following reduced game, to be denoted the 1-reduced game;
$vT,x(T)=x(T)$

$$v_{T,x}(S) = \sum_{k=0}^{|N\setminus T|} \alpha_k \sum_{\substack{R\subseteq N\setminus T\\ |R|=k}} [v(S\cup R) - x(R)]$$

$v_{T,x}(\emptyset)=0$

$$v_{T,x}(S)=\alpha_{|N\setminus T|}[v(S\cup N\setminus T) - x(N\setminus T)] + \dots + \alpha_{|R|} \sum_{\substack{R\subseteq N\setminus T\\ |R|=k}} [v(S\cup R) - x(R)] + \dots + \alpha_0 v(S),$$

where $\alpha\in R^{|N\setminus T|}$ is a vector satisfying $0<\alpha_i<1$ for all i and $\sum\alpha_i=1$. In other words, the 1-reduced game is a convex combination of reduced games of the form $\sum [v(S\cup R) - x(R)]$ where the sum runs over all coalitions $R\subset N\setminus T$ of fixed size. It is natural to wonder about the significance of these reduced game terms.

Definition 1.1: Let the k-marginal of player i be defined to be the average worth of coalitions of cardinality k {1,2,...,n-1} containing i to the grand coalition, ie.

$$s_i= v(N)- \binom{n-2}{k-1}^{-1} v(R), \text{ sum over all } R\ni i,\ |R|=k$$

$$s_i = v(N) - \binom{n-2}{k-1}^{-1}\sum v(R)$$

Then the Equal Allocation of k-marginal Value is the allocation method which first allocates to each player her k-marginal worth si, and the amount remaining is divided equally among the players, ie.,

$$EAkV_i= s_i + 1/n[v(N) -\sum s_j]$$

Remark 1.2: The EAkV formula can be equivalently written as

$$EAkV_i= \binom{n-2}{k-1}^{-1}\sum_{\substack{|R|=k\\ i\in R}} v(R) + 1/n\Big[v(N) - k\binom{n-2}{k-1}^{-1}\sum_{|R|=k} v(R)\Big]$$

Examples: Both the equal allocation of nonseparable value (k=n-1) and the equal allocation of joint value (k=1) are examples of EAkV's.

Theorem 1.3: The EAkV is the unique allocation method which is standard on two player games and which is consistent on the following reduced game:

$v_{T,x}(T)=x(T)$ $\quad$ $v_{T,x}(\emptyset)=0$

$$v_{T,x}(S)= \binom{|T^c|}{k}^{-1}\sum[v(S\cup R) - x(R)], \text{ sum over all } R\subset N-T,\ |R|=k-1$$

proof: we will use the notation $v_T(S)= v_{T,x}(\{S\})$ when the meaning is clear.

existence -

The only EAkV defined on two player games is the EAJV must have k<n). Simple substitution of the values yields the

standard formula on two player games. Alternatively, the proof could proceed from the point of marginals (EANV could be the fundamental allocation method, but for clarity and simpler formulae I will assume EAJV fund. However, interpretation dictates EANV to be fundamental) The proof of existence involves three assertions; 1. EAkV is defined to be EAJV (resp EANV) on games for which k >= t, and thus there are two cases to consider demonstration of consistency; 2. k<t (EAkV reduced game applied to EAkV formula) and 3. k>t (EAkV reduced game applied to EAJV formula). The proof would then be a substitution of T for N and vT for v in the corresponding formula and working out the simplification, using vT(T)= x(T).

Given an arbitrary game (N,v) and a coalition T, |T|=t, the value of EAkVi on the reduced game (T,vT) is the following:

EAkVi= vT(R)+
subsumed in EAJV (EANV resp.) two cases to consider.
Actually EAJV is basic ie. n=1 then simplex is v(S) (single point, n-1 dim simplex)

uniqueness -

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We actually show a slightly stronger statement, namely that standard on two person games and 2-player EAkV-consistency imply a unique solution.

Consider the coalition T={1,i}. Let be an allocation method which is standard on two player games and EAkV-consistent. Then we have the following equalities:

$$\theta i(N,v)= \theta i(T,vT)= vT(i)+ (1/2)(vT(\{1,i\})- vT(1)- vT(i))$$
$$= (1/2)(vT(1i)+ vT(i)- vT(1))$$
$$= (1/2)(i(T,vT)+ 1(T,vT)+ vT(i)- vT(1))$$

$$\Rightarrow \theta i= \theta 1+ vT(i)- vT(1)$$

Substituting in the formula for vT,θ and simplifying the expression yields

$$\theta i= \theta 1+ \binom{n-2}{k}^{-1}[v(i\cup R)- v(1\cup R)] \qquad \sum_{R\subseteq N-T\ |R|=k}$$

Note that the same argument applies to any i=2 to n, thus generating n-1 linear conditions on . Applying efficiency, and substituting the values for 2,..., n from above, we get

$$n\theta 1+ (r+1)\binom{n-2}{k}^{-1}\sum_k [v(R)]- (n-r-1)\binom{n-2}{k}^{-1}\sum_k [v(R)]= v(N)$$

$$\Rightarrow \theta 1= (1/n)v(N)+ (n-r-1)/n\binom{n-2}{k}^{-1}\sum_k [v(R)]- (r+1)/n\binom{n-2}{k}^{-1}\sum_k [v(R)]$$

which is easily seen to be identical to the alternate form of the solution given in Remark 1.2!

Corollary 1.3.1: The EAJV is the unique allocation method which is standard on two player games and which is consistent on the following reduced game:

$vT,x(T)=x(T)$

$vT,x(S)=v(S)$ for all $S \subsetneq T$

Corollary 1.3.2:(Moulin,1984) The EANV is the unique allocation method which is standard on two player games and which is consistent on the following reduced game:

$vT,x(S)= v(S \cup N-T) - x(N-T)$ for all $S \subset T$

$vT,x(\emptyset)=0$

Remark: The uniqueness proof for these two corollaries is an easy exercise.

Aside: Three possible economic situations for the EAkV allocation method set

The essential concept needed to understand the EAkV solutions intuitively is the notion of relative utility of coalitions of given size. Given a particular k<n, the EAkV solution is the extreme notion that the coalitions of size k are the only important coalitions relative to the grand coalition. However, though extreme, there are plausible economic situations where the EAkV solution may be a reasonable allocation. For example, consider a factory with N>>3 workers. The product being assembled requires teams of

three workers. More than three workers lowers the productivity of the team (too many cooks spoil the broth). Thus the only important coalition size is coalitions of size three. Note, however, that coalitions of other sizes have nonzero values (thus making application of the Shapley value questionable as coalitions of other sizes will be less likely to form). Each worker has a certain amount of skill, experience, and ability to work with his/her coworkers. Thus there are varying values for various coalitions. The value of the grand coalition to be divided among the workers is of course the profit of the factory, ie. player i's assignment is his/her salary.

In general, the raw EAkV allocation method seems to have most application to socialist economic systems. However, more complex allocation methods can be constructed from these simple allocation methods. For example, Maltenfort (1990) classified a set of linear allocation methods which satisfy a weak form of the dummy player property. Formally, the set can be characterized axiomatically by (efficiency,) symmetry, linearity, and the inessential game property. is linear if, for all a,b R,

$$a\theta(N,v) + b\theta(N,w) = \theta(N,av) + \theta(N,bw) = \theta(N,av+bw)$$

where av and bw are the value functions defined by (av)(S)=av(S) and (bw)(S)=bw(S) for all $S \subset N$. A game (N,v) is inessential if, for all $S \subset N$, v(S)= v(i), sum over $i \in S$. has the inessential game property if θ i(N,v)=v(i) for all inessential games (N,v).

Lemma 1.4.1: Every element of the Maltenfort set can be uniquely expressed as a convex combination of EAkV's

proof: EAkV's satisfy the four axioms. The only nontrivial axiom to check is inessential game proprerty. Proof for that axiom is straightforward (Simple) Conversely, Maltenfort solution => lin comb. EAkV's Easily shown by comparing the two formulae.

Corollary 1.4.1: The Shapley value is the average of the EAkV's

proof: It is an easy exercise to show that

SHi=(n-1)-1Σ EAkVi, sum over k=1 to n-1

New interpretation of the Shapley value (average k-marginal value)

Corollary 1.4.1 highlights an interesting, possibly new interpretation of the Shapley value, ie. the average k-marginal value of player i. There is some basis for this interpretation in Shapley's own thoughts about the Shapley value. According to Harsanyi (1977, p. 226), Shapley originally conceived of the Shapley value as the average payoff prospect for player i, ie, the average solution. In other words, if the game were played many times, the negotiated solutions would likely cluster about the Shapley

value. Thus, the Shapley value is not necessarily a viable solution in its' own right, but simply a statistical measure of classical game outcomes. This analysis suggests an interpretation for the general Maltenfort solution as the weighted average payoff prospect, ie. the average solution after accounting for the importance of the size of the coalition. In fact, the Shapley value is calculated over an excess space for which the excesses are normalized, ie. a coalition of size s has an excess weighted by %$#$@. Thus, the Shapley value is more closely aligned with the per capita nucleolus than with the nucleolus (calculated over the same excess space)

The interpretation of the Shapley value reduced game is motivated by a reexamining of the interpretation of the Shapley Value. Shpley originally thought of his value as the average payoff prospect given a large number of iterations of the bargaining process.

Theorem 1.4: Any element of the Maltenfort set is uniquely axiomatically characterized by symmetry, covariance and 1-consistency for some $\alpha \in R^n$. Further, every element of the Maltenfort set can be so characterized.

existence -

The problem with following the existence method for Corollaries 1.2.1 and 1.2.2 is the following: When applying the consistency property to a general combination of reduced games, all elements of the reduced game must be applied to all parts of the reduced game, eg the EAJV reduced game is applied to EANV and vice versa. It does not appear that these pieces cancel out, or create something comprehensible. (However, see proof of EAKV consistency for possible solution)

uniqueness as before 2player 1-consistency

The uniqueness proof is, mutatis mutandis, identical to the proof for the uniqueness of the EAkV's. (Given N players, need convex comb of first n-1 EAkV's)

Corollary 1.4.2:(Sobolev,1973 - see Dreissen, 1990) The Shapley value is the unique allocation method satisfying symmetry, covariance, and consistency with respect to the following reduced game (given for $T=N\setminus i$):

$$v_{N\setminus i,x}(S) = (n-1)^{-1}|S|[v(S \cup \{i\}) - x_i] + (n-1)^{-1}(n-|S|-1)v(S)$$

Remark: Note that the n-1 case of the reduced game uniquely determines the value of the reduced game for all T < n-1

<u>The Weighted Prenucleolus</u>

Digression: Though much work has been done with the nucleolus as defined by Schmeidler (1969) and its extension to the space of all games, the prenucleolus, relatively

little work has been done with related allocation methods. For example, the per capita nucleolus, as defined by Grotte (1970), and its generalization to the space of all games, the per capita prenucleolus, remain relatively unused. This is unfortunate as these allocation methods can have several interesting properties, for example, the per capita (pre)-

nucleolus is both aggregate monotone and group rational (Housman, 1990).

Definine the following reduced game, to be denoted the w reduced game;

$vT,x(T)=x(T)$

$vT,x(\emptyset)=0$

$vT,x(S)= x(S) + \max_{R\subset N-T}\{(w|S|)-1[e(S\cup R,x,v)]\}$ for all $S\subset T$

where $e(S,x,v)=w|S|[v(S) - x(S)]$ is the excess of coalition S in game (N,v).

Lemma 2.1.1: $e(S,x,vT,x)= \max_{R\subset N-T}\{e(S\cup R,x,v)$

proof: $e(S,x,vT,x) = w|S|[vT,x(S) - x(S)]$

$$= w|S|[x(S) + \max_{R\subset N-T}\{(w|S|-1)[e(S\cup R,x,v)]\} -x(S)$$

$$= w|S|[(w|S|-1\max_{R\subset N-T}\{e(S\cup R,x,v)\}]$$

$$= \max_{R\subset N-T}\{e(S\cup R,x,v)\}$$

Theorem 2.1: The weighted (pre)nucleolus is consistent with respect to the w reduced game.

proof : Let x be the weighted (pre)nucleolus.
Let $C_\alpha = \{S\subset T : e(S,x,vT) > \alpha\}$.
Consider $\varsigma_\alpha = \{S\subsetneq N : e(S,x,v) > \alpha\}$. Since x is the weighted (pre)nucleolus, ς_α is balanced with respect to δ when nonempty by Kohlberg's criterion (ie. property II. See (Kohlberg,1970) or (Grotte,1970)). This implies that $\varsigma_\alpha\cap T = \{S\cap T : S\in\varsigma_\alpha\}$ is balanced on T with balancing weights $\delta(S) =\sum\delta(S)$, $S\in\varsigma_\alpha$, $S\cap T=R$. Thus, it only need be shown that $C_\alpha = \varsigma_\alpha\cap T$. But this is obvious by Lemma 2.1.1, as follows;

$Q\in C_\alpha \iff e(Q,x,vT) > \alpha \iff e(Q\cup R,x,v) > \alpha$ for some $R\subsetneq N-T$
$\iff Q\cup R\in\varsigma_\alpha$ for some $R\subsetneq N-T$
$\iff Q\in\varsigma_\alpha\cap T$!

Corollary 2.1.1: The per capita prenucleolus is consistent with respect to the following reduced game:

$vT,x(T)=x(T)$

$vT,x(\emptyset)=0.$

$$vT,x(S)= x(S) + \max_{R\subset N-T}\{|S|-1[e(S\cup R,x,v)]\}$$

The General Reduced Game Form and some Economic Interpretations
The General Reduced Game Form:

$vT,x(T)= x(T)$

$vT,x(\emptyset)= 0$

$vT,x(S)= x(S) + \{e(S \quad R,x,v): R\subset N-T\}$, $S\subset T$, where

$$e(S\cup R,x,v)=w|S \cup R|[v(S\cup R) - x(S\cup R)].$$

In words, an allocation's reduced game is determined by two things; the excess space (the basis chosen), and the function which takes the excess space as its argument. The excess space is determined by specifying the relative utility value of coalitions of a particular weight. Actually is Rn, different weights correspond to different choice of basis. canonical basis is simple excesses.

The value of an arbitrary subcoalition S is simply the allocation that the players in S receive in the large game plus the coalition's complaint with the allocation. (Complaint is a measure of how likely the coalition will prefer not to join the large game) Note that a complaint implies that the complainers find something unfair about the allocation. Thus the function is determined by the particular property of fairness to which the players of the game are appealing. (The distinctive notion of fairness differentiating one allocation method from others is encapsulated in the choice of .) For example, the (pre)nucleolus reduced game function is the maximization function (maximum excess to which the players can lay claim, as is obvious by Theorem 2.1), while the Shapley value reduced game function is the summation function (linear combination of excesses corresponding to the . general form -159, allocation plus "excess",show special cases satisfy, Note that the weights of the excesses must be stipulated in order to specify the allocation method.

Remark: The w reduced game has an interesting economic interpretation. A reduced game can be thought of as a test of the "fairness" of an allocation x by the players in the coalition T. When playing a reduced game, we assume that the players not in T are satisfied with the allocation x. The reduced game for an arbitrarily subcoalition S T is the assignment of x(S) plus a measure of the complaint that is converted to size |S| utility.

Since the complaints are weighted according to the size of S, the complaint must be converted to the utility of the subcoalition S.

w|S| and i's are the same thing. Allocation method specifies weights. Restrictions on to be convex for sol'n. Maximization implies that proportional invariance (all weights equivalent to a convex combination. equivalence relation if produce same maximization. Same max if proportional weights. (Fix game)) ie similar simplexes are equivalent? general case? w|S| and measure relative utility of coalition of size |S|.

Questions:
The most pressing item to be solved is the following
Hypothesis : The weighted prenucleolus is uniquely defined by symmetry, covariance and w-consistency.
The following Corollaries will then be immediate.
Corollary: (Sobolev) The (per capita) prenucleolus is the unique allocation method satisfying symmetry, covariance, and w-consistency.
proof let w|S|=1 (w|S|=1/i per capita case) for all S

Reduced axioms need for w-prenuc characterization? (standard?)
Though I have assumed symmetry and covariance throughout this paper, it is by no means clear that these are the weakest axioms needed. Perhaps weaker properties, involving the standard solution on two person games, could also characterize the allocation method classes presented in this paper.

hart & mas-colell? results with the egalitarian value as well as use of x vs. (Thomson) .Discuss two differences One way gives EANV(My first result, ind dis by Moulin) but other way not help (Max is Sh reduced set of games)Then it suggests that the h&MC approach is a different concept entirely. Equivalent method of defining consistency seems to be possible? Or only possible for reduced set of allocation methods?
Note that the reduced games presented in this paper are not dependant on an allocation method. The reduced game can be calculated for any allocation. This approach can be generalized to classify allocations in general. However, the H&MC approach specifically requires an allocation method to calculate the reduced game (thus a more complicated and possibly more specialized concept than heretofore discussed)

other allocation methods? eg,tau value? (PAJV?)
The question naturally arises whether other allocation method reduced games fit this general form. Though this question is valid for set-theoretic allocation methods, the

axiomatic descriptions involving consistency of these solution concepts have involved only the Davis and Maschler reduced game, which has already been demonstrated to be of the general form. The function in the general reduced game need only be replaced by a relation to achieve full generality, thus incorporating all reduced game descriptions in the literature not involving H&MC consistency. (See below) However, one functional allocation method which may shed new light is the Tau value. I fee that it would be worthwhile to explore the connection between the Tau value and the Proportional Allocation of Joint Value and it's generalizations. I have a hunch that the Tau value reduced game is some function of generalized excesses which generate the Poportional Allocations.

query: red game is 1 red game, i=1/n for all i. Sh to nuc. A functional allocation method's reduced game is a finite valued function on the n-1 dimensional simplex of a chosen basis of weighted excesses. Proof for Shapley value generalization is trivial. Proof for weighted nucleolus: Initially power set dimensional, n-1 dimensional implied by symmetry, proportionality of weights implied by covariance (use normalized weights or define to be noormalized. Multiplication by inverse sum of weights is irrelevant by proportionality. Addition of x(S) irrelevant by value separability.) implies convexity which implies simplexity.

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