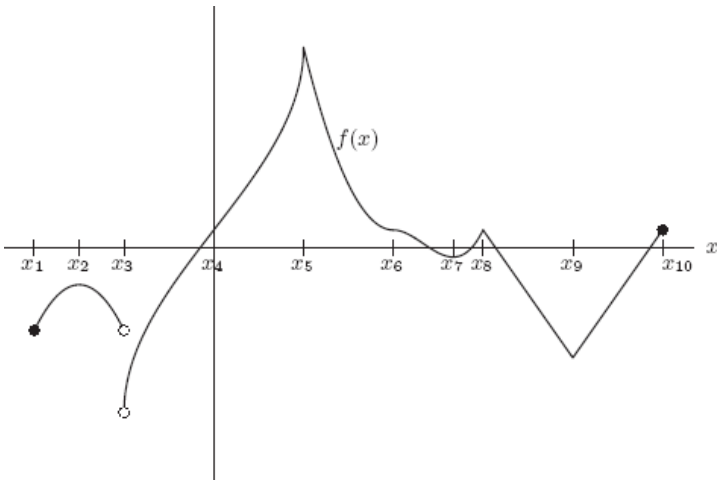


## Critical Points, Local Maxima, and Local Minima

Local maximum?



What are the critical points of  $f$ ?

What are the local minima of  $f$ ?

What are the local maxima of  $f$ ?

How do the above answers change if the upper hole at  $x_3$  is filled in?

How do the above answers change if the lower hole at  $x_3$  is filled in?

The table records the rate of change of air temperature,  $H$ , as a function of hours since midnight,  $t$ , during one morning.

$t$	6	7	8	9	10	11	12
$dH / dt$	1	2	0	-2	0	3	2

When was the temperature a local minimum?

## Critical Points, Local Maxima, and Local Minima

Find all critical points, local minima, and local maxima of the following functions.

$$f(x) = 4x^3 + 3x^2 - 36x - 5$$

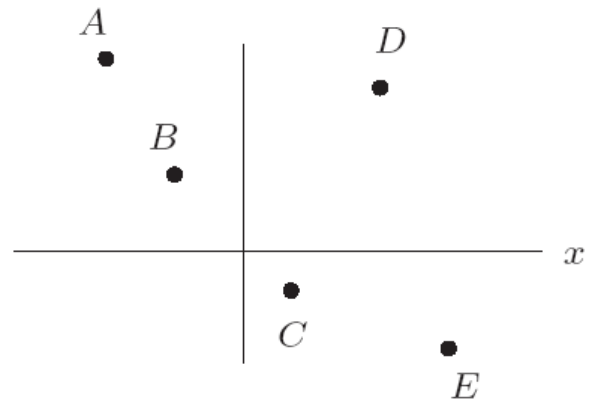
$$g(x) = x - 2\ln(x^2 + 3)$$

## Critical Points, Local Maxima, and Local Minima

Find a differentiable function that has a local maximum at  $(0,5)$ , a local minimum at  $(2,1)$ , and no other local extrema.

Graph two continuous functions  $f$  and  $g$ , each of which has exactly five critical points, the points A-E in Figure 4.12, and which satisfy the following conditions:

- (a)  $f(x) \rightarrow \infty$  as  $x \rightarrow -\infty$  and  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$   
 (b)  $g(x) \rightarrow -\infty$  as  $x \rightarrow -\infty$  and  $g(x) \rightarrow 0$  as  $x \rightarrow \infty$



Which of the following pieces of information from a daily weather report allow you to conclude with certainty that there was a local maximum of temperature at some time after 10:00 am and before 2:00 pm?

- (a) Temperature  $50^\circ$  at 10:00 am and  $50^\circ$  and falling at 2:00 pm.  
 (b) Temperature  $50^\circ$  at 10:00 am and  $40^\circ$  at 2:00 pm.  
 (c) Temperature rising at 10:00 am and falling at 2:00 pm.  
 (d) Temperature  $50^\circ$  at 10:00 am and 2:00 pm,  $60^\circ$  at noon.  
 (e) Temperature  $50^\circ$  at 10:00 am and  $60^\circ$  at 2:00 pm.

Assume  $f$  has a derivative everywhere and just one critical point, at  $x = 3$ . In parts (a) – (d), you are given additional conditions. In each case, decide whether  $x = 3$  is a local maximum, a local minimum, or neither. Sketch possible graphs for all four cases.

- (a)  $f'(1) = 3$  and  $f'(5) = -1$   
 (b)  $f(x) \rightarrow \infty$  as  $x \rightarrow \infty$  and as  $x \rightarrow -\infty$   
 (c)  $f(1) = 1, f(2) = 2, f(4) = 4, f(5) = 5$   
 (d)  $f'(2) = -1, f(3) = 1, f(x) \rightarrow 3$  as  $x \rightarrow \infty$