

1. $f(x)$ is the age of Antarctic ice (in hundreds of years) at a depth of x meters below the surface.
- (a) In words, what is the practical meaning of $f(10)$?
 $f(10)$ is the age of the ice, 10 meters below the surface.
- (b) Is f increasing or decreasing, and why?
 You would expect f to be increasing, because the deeper you go, the older the ice.

2. From the following table

Table 1.1.1

| | | | | | | |
|--------|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| $f(x)$ | 2 | 3 | 7 | 6 | 4 | 2 |

- (a) Find $f(3)$. $f(3) = 7$
- (b) Find the value(s) of x that give $f(x) = 2$. $f(x) = 2$ when $x = 1$ or when $x = 6$
3. An object is put outside on a cold day and its temperature, H , in degrees Celsius, is a function of the time, t , in minutes since it was put outside.
- (a) What does the statement $H(30) = 10$ mean? Use words and remember to include units in your answer.
 30 minutes after the object is put outside, its temperature is 10 degrees Celsius

- (b) The graph of H versus t is shown below. Explain in terms of temperature of the object and the time outside, what each of the following mean.

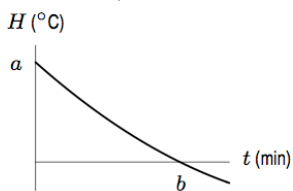


Figure 1.1.3

- i. vertical intercept a
 The vertical intercept a is the temperature when the object is first placed outside.
- ii. horizontal intercept b
 The horizontal intercept b is the time it takes for the object to reach 0 degrees Celsius.
4. Suppose $g(x)$ is an exponential function. Complete the table of values for the function g below.

| | | | | | |
|--------|----|----|----|----|----|
| x | 0 | 5 | 10 | 15 | 20 |
| $g(x)$ | 10 | 20 | ? | ? | ? |

Now find a formula for $g(x)$. Various equations (all equivalent) are possible - One simple one is $g(x) = 10(2)^{x/5}$

5. Values for $g(x)$ are given in the table below. Is $g(x)$ concave up, concave down, or neither?

Table 1.3.9

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|--------|-----|----|----|----|----|----|
| $g(x)$ | 100 | 90 | 81 | 73 | 66 | 60 |

6. Sketch a graph of a function that is decreasing at an increasing rate.

[I've decided this is an ambiguous question. Instead:]

Sketch a function that is increasing at an increasing rate: It's concave up:



7. A population is growing according to the function $P = 250(1.065)^t$, where P is the population at time t .

(a) What is the initial population? 250

(b) What is the annual growth rate? 6.5%

(c) What is the population in year 10? Compute $250(1.065)^{10}$

(d) How many years will it take for the population to reach 1000? Solve for t . $1000 = 250(1.065)^t$. So $4 = (1.065)^t$. Take natural log of both sides: $\ln(4) = t \ln(1.065)$. Therefore $t = \ln(4)/\ln(1.065)$. So about 22 years.

8. An exponentially decaying substance was weighed every hour and the results are given below. If the

formula $Q = Q_0 e^{-kt}$ gives the weight of the substance, Q , at time t in hours since 9 am, then

$Q_0 =$ 14 and $k =$ 0.11. Round k to 2 decimal points.

| Time | Weight (in grams) |
|---------|-------------------|
| 9 am | 14 |
| 10 am | 12.542 |
| 11 am | 11.235 |
| 12 noon | 10.065 |
| 1 pm | 9.017 |

9. A bakery has 200 lbs of flour. If they use 5% of the available flour each day, how much do they have after 10 days? How much do they have left after n days?

After 10 days they have about 119.7 lbs

The formula is $A = 200 \cdot (0.95)^n$ where A is the amount of flour after n days.

10. If $8 \cdot (2.5^x) = a \cdot e^{kx}$ find a and k .

$a = 8$ and $e^k = 2.5$ so $k = \ln(2.5) = 0.916$

11. If the size of a bacteria colony doubles in 8 hours, how long will it take for the number of bacteria to be 5 times the original amount?

If the size doubles in 8 hours, we can write this as $N = N_0 (2)^{t/8}$. Solve $5 N_0 = N_0 (2)^{t/8}$.
 Or $\ln(5) = t/8 \ln(2)$. So $t = 8 \ln(5) / \ln(2) = 18.57$ hours - so a little over 18 and a half hours.

12. A cigarette contains about 0.4 mg of nicotine. The half-life of nicotine in the body is about 2 hours. How long does it take after smoking a cigarette, for the level of nicotine in a smoker's body to be reduced to 0.08 mg?

If the half life is 2 hours we can write this as $A = 0.4 (1/2)^{t/2}$. Solve $0.08 = 0.4 (0.5)^{t/2}$.
 Or $\ln(0.08/0.4) = t/2 \ln(0.5)$. So $t = 2 \ln(0.08/0.4) / \ln(0.5) = 4.64$ hours - so about 4 hours and 40 minutes.

13. Use the table below.

Table 1.8.19

| | | | | | |
|--------|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| $f(x)$ | 2 | 4 | 6 | 3 | 5 |
| $g(x)$ | 5 | 3 | 2 | 1 | 0 |

Find $f(g(1)) = 3$ $g(f(1)) = 0$ $f(g(3)) = 4$ $g(f(3)) = 1$

14. The graph of $y = f(x)$ is shown below.

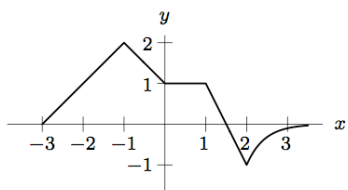
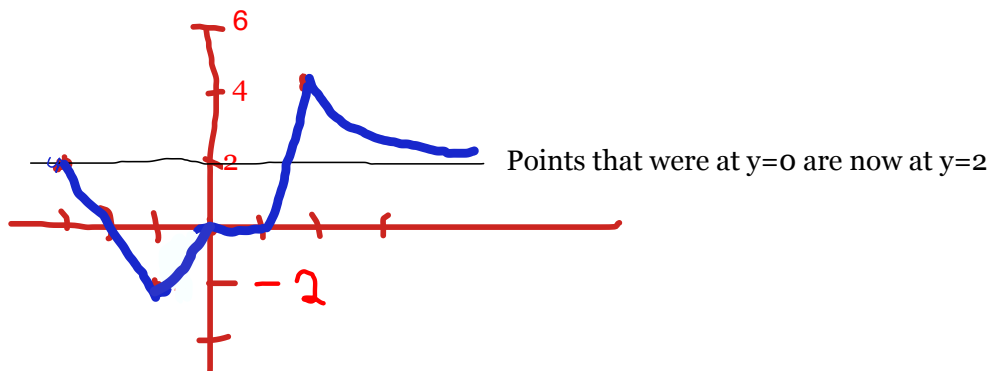


Figure 1.8.23

Sketch the graph of $y = 2 - 2f(x)$. Stretch by 2 vertically, flip it over the horizontal axis, Shift up by 2



15. The number of species S on an island is proportional to the square root of the area A of the island. An island with an area of 4 square miles contains 20 species.

(a) Find a formula for S as a function of A .

$S = k \cdot \sqrt{A}$. Since we know that when $A = 4$ that $S = 20$, we can solve for k . Set up $20 = k \cdot \sqrt{4}$. So $20 = 2k$, $k = 10$. So our equation is $S = 10 \cdot \sqrt{A}$

(b) If an island is 9 square miles in area, determine the number of species expected on the island.

Compute $S = 10 \cdot \sqrt{9} = 10 \cdot 3 = 30$

16. Consider the function given in the table below.

Table 1.10.22

| | | | | | | | | | |
|--------|----|---|----|----|----|---|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| $f(x)$ | -2 | 0 | -2 | -4 | -2 | 0 | -2 | -4 | -2 |

(a) Explain why the function represented in the following table appears to be periodic.

The numbers in the $f(x)$ column start to repeat.

(b) Approximate the period and the amplitude of the function.

period 4, amplitude 2

(c) Assuming the function is periodic, estimate $f(10)$ and $f(15)$.

So $f(10)$ would be the same as $f(6)$ and $f(15)$ would be the same as $f(11)$ which is the same as $f(7)$.

17. Find an equation which defines the function shown below. Period 8π and amplitude is 2. The midline of the function is at $y=2$. The equation would be

$$y = 2 \cdot \sin\left[\frac{2\pi}{8\pi}x\right] + 2 = 2 \cdot \sin\left[\frac{1}{4}x\right] + 2$$

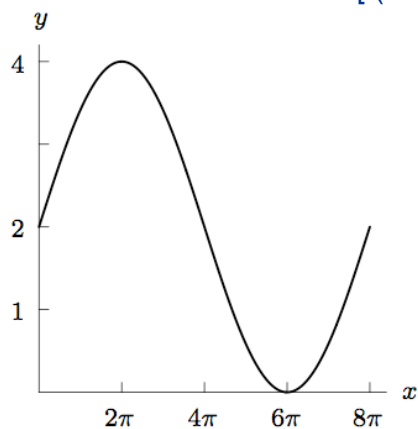


Figure 1.10.31