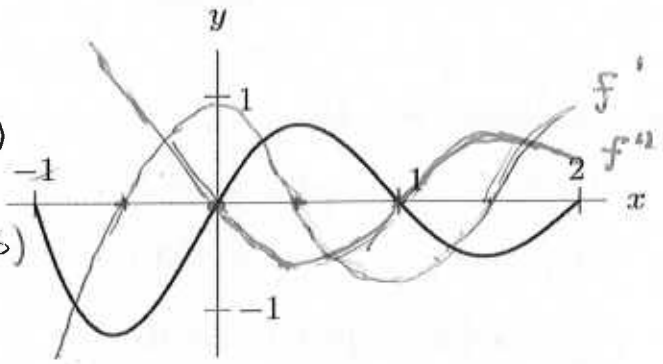
Math 211, Exam 3 Review Exercises ANSWERS 11/20/2013

1. The graph of $f(x)$ is given in the figure to the right.



   (a) For which values of x is f increasing? $(-.5, .5) \cup (1.5, 2)$

   (b) For which values of x is f decreasing? $(-1, -.5) \cup (.5, 1.5)$

   (c) For which values of x is f concave up? $(-1, 0) \cup (1, 2)$

   (d) For which values of x is f concave down? $(0, 1)$

   (e) Where does $f'(x)$ change its sign? $x = -.5, .5, 1.5$

   (f) Where does $f'(x)$ have a local maximum or minimum? -1 (min) 0 (max) 1 (min) 2 (max)

   (g) Sketch the graph of $f'(x)$ on the same axes. in blue

   (h) Sketch the graph of $f''(x)$ on the same axes in red

2. Find constants a and b in the function $f(x) = axe^{bx}$ such that $f(1/3) = 1$ is a local maximum.

$f'(x) = ae^{bx} + abxe^{bx}$

$= ae^{bx}(1+bx)$

$f'(x) = 0 \Leftrightarrow a = 0$ or $1+bx = 0$

$x = -1/b$

If $b = -3$, then $1/3$ is a critical point.

$f(1/3) = a(\frac{1}{3})e^{-1} = 1 \Rightarrow a = 3e$

x		$1/3$	
$f'(x)$	+	0	−

Hence, $1/3$ is a local max

3. The number of plants in a terrarium is given by the function $P(c) = -1.2c^2 + 4c + 10$ where c is the number of mg of plant food added to the terrarium. Find the amount of plant food that produces the highest number of plants.

$P'(c) = -2.4c + 4$

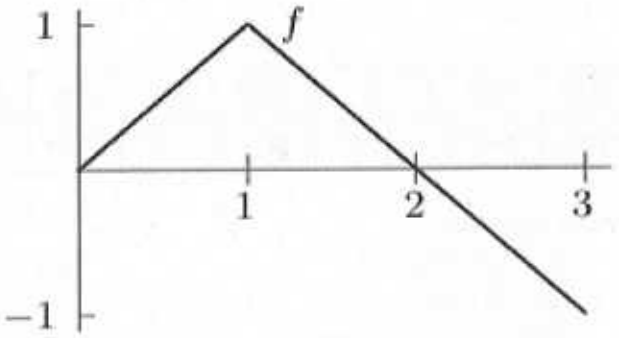
c is a critical point iff

$0 = P'(c)$ iff $c = 4/2.4 = 1.67$

$P''(c) = -2.4 \Rightarrow$ critical point is a max.

The amount of food is 1.67 mg.

4. The graph of f is shown below. If $F' = f$ and $F(0) = 3$, then what is $F(3)$?



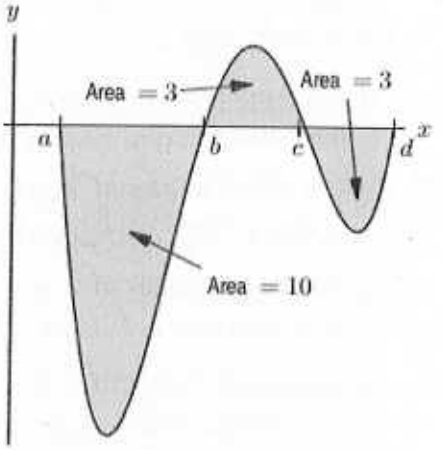
$F(3) = F(0) + \int_0^3 f(t)\,dt$

$= 3 + 1 - \frac{1}{2}$

$= 3.5$

5. Be able to find antiderivatives such as those on page 322: 1-41 in your text.

6. The graph of f is shown to the right.



Estimate

$\int_a^b f(x)dx = -10$

$\int_a^c f(x)dx = -10+3 = -7$

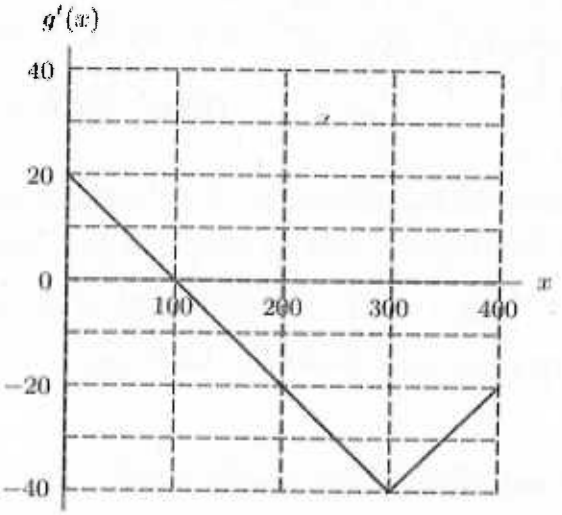
$\int_a^d f(x)dx = -7-3 = -10$

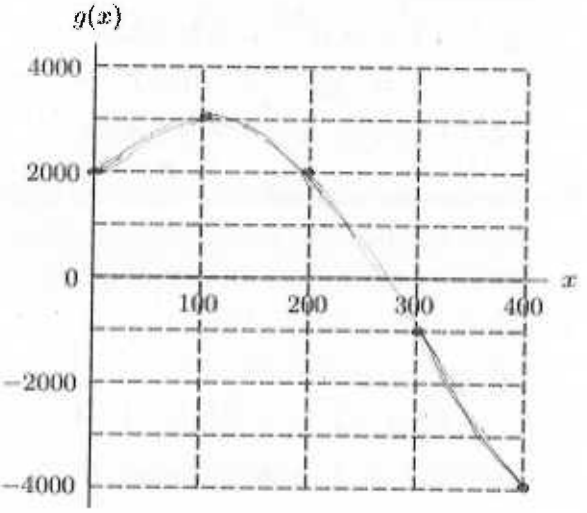
$\int_a^d |f(x)|dx = 10+3+3 = 16$

7. Use the fundamental theorem of calculus to determine the value of b if the area under the graph of $f(x) = 3x^2 + 1$ between $x = 0$ and $x = b$ is 30. Assume $b > 0$.

$30 = \int_0^b (3x^2+1)dx = x^3+x\Big|_0^b = b^3+b \Rightarrow b = 3$

8. The graph of $g'(x)$ is shown. Sketch the graph of $g(x)$ assuming $g(0) = 2000$.





Then fill in the table below.

x	0	100	200	300	400
$g(x)$	2000	3000	2000	−1000	−4000

x	100	200	300	400
$g'(x)$	20	−20	−40	−20

Determine if the following are positive or negative.

(i) $g(50)$ +
(ii) $g(150)$ +
(iii) $g(350)$ −
(iv) $g'(50)$ +
(v) $g'(150)$ −
(vi) $g'(350)$ −
(vii) $g''(50)$ −
(viii) $g''(150)$ −
(ix) $g''(350)$ +