Limits, Continuity, and the Definition of Derivative

Chapter 2 Focus on Theory (pages 127-132)

An Example

Suppose the distance (in light-years) of the Starship Enterprise from the Earth at t years after its mission started is given by $f(t) = 8\sin\left(\frac{\pi}{5}t\right) + \sin(7\pi t)$.

1. How far from the Earth is the Starship Enterprise 2.1 years after its mission started?
2. What was the average speed (with respect to the Earth) of the Starship Enterprise during the first 2.1 years of its mission?
3. What was the instantaneous speed (with respect to the Earth) of the Starship Enterprise 2.1 years after its mission started?

Limits

Definition. If the values of $g(x)$ get closer and closer to a finite number as x gets closer and closer to a, we call that finite number the *limit* of $g(x)$ as x approaches a, and denote it by $\lim_{x\to a} g(x)$.

1. What is the limit of $g(x) = e^{-\tan(\pi x)}$ as x approaches 0.3?
2. What is the limit of $g(x) = e^{-\tan(\pi x)}$ as x approaches 0.5?
3. What is the limit of $h(x) = \cos\left(\frac{\pi}{x}\right)$ as x approaches 0?
4. What is the limit of $\frac{f[2.1+\Delta t]-f[2.1]}{\Delta t}$ as Δt approaches 0?

Continuity

Definition. A function $g(x)$ is *continuous* at $x = a$ if $\lim_{x\to a} g(x) = g(a)$.

Remark. Implicit in this statement are three criteria: (1) the function g(x) must be defined at a, (2) the limit of f(x) as x approaches a must exist, and (3) the two must be equal.

1. Is $g(x) = e^{-\tan(\pi x)}$ continuous at $x = 0.3$?
2. Is $g(x) = e^{-\tan(\pi x)}$ continuous at $x = 0.35$?
3. Is $\frac{f(2.1+\Delta t)-f(2.1)}{\Delta t}$ continuous at $\Delta t = 0$?

Derivatives Analytically

1. If $f(x) = x^2 - 4x + 7$, find $f'(3)$.
2. If $f(x) = x^2 - 4x + 7$, find $f'(x)$.