Math 213 - Tabular Data

Wave Heights on the Open Sea

The wave heights h in the open sea depend on the speed v of the wind (knots) and the length of time t that the wind has been blowing at that speed (hours). Values for the function $h = f(v,t)$ are in the following table.

$t=15$ $t=20$

v\t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

$v=40$

Questions:

1. What is the value of $f(40,15)$? What is its meaning?

If a 40-knot wind has been blowing for 15 hours, the ocean waves will have a height of 25 (feet?)

$h(40,15) = 25$ (feet?)

2. What is the meanings of the function $h = f(30,t)$? $h = f(v,30)$?

f(30,t) is the height of waves as a function of t (how long the wind has been blowing at 30 knots).

3. Estimate the values of $\frac{\partial f}{\partial v}(40,20)$ and $\frac{\partial f}{\partial t}(40,20)$ and interpret their meanings.

$\frac{\partial f}{\partial v} \approx \frac{\Delta f}{\Delta v} \approx \frac{40-17}{50-30} = \frac{23 \text{ feet}}{20 \text{ knots}} \approx 1.15 \text{ ft/knot}$

When the wind has been blowing for 20 hours, for every 1 knot increase in wind speed, the height of waves increases by 1.15 foot.

$\frac{\partial f}{\partial t} \sim \frac{\Delta f}{\Delta t} = \frac{31-25}{30-15} = \frac{6 \text{ ft}}{15 \text{ hrs}} = 0.4 \text{ ft/hr}$

When the wind is blowing at 40 knots, the wave height increases by 0.4 ft for each additional hour that it blows.

4. Find a linear approximation to the wave height function when v is near 40 knots and t is near 20 hours. (Round the numerical coefficients to two decimal places).

$h(v,t) = \underbrace{28}_{h(40,20)} + \underbrace{1.15}_{\left.\frac{\partial h}{\partial v}\right|_{(40,20)}}(v-40) + \underbrace{0.4}_{\left.\frac{\partial h}{\partial t}\right|_{(40,20)}}(t-20)$

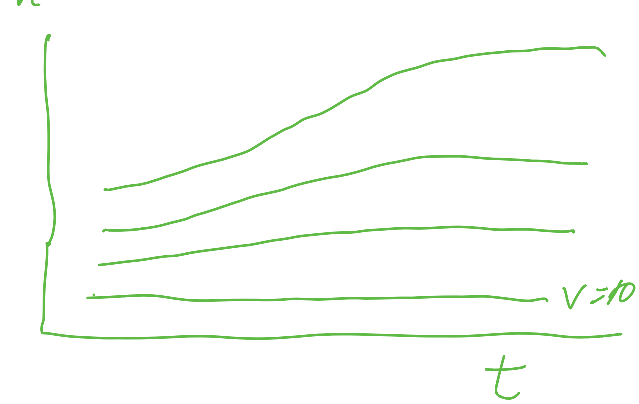
5. Using the linear approximation, estimate the wave heights when the wind has been blowing for 24 hours at 43 knots. (Round the answer to two decimal places).

$h(43,24) = 28 + 1.15(3) + 0.4(4) = 33.05$ (feet)

6. What do you think is the $\lim_{t\to\infty} \frac{\partial f}{\partial t}$?

It seems as if each of the rows is "saturating", eventually, at large t values, reaching a constant height. That is

$\lim_{t\to\infty} \frac{\partial f}{\partial t} = 0$



Math 213 - - Tabular Data

Partial Derivatives and Data

The function $f(x,y)$ is given by the following data.

	x=0	x=10	x=20	x=30
y=0	89	80	74	71
y=2	93	85	80	76
y=4	98	91	85	81
y=6	104	98	92	88
y=8	112	105	99	94

What is $f(10,6)$? $= 98$

If $f(x,y)=98$ and $y=4$ then what is x? $x=0$

Estimate $\dfrac{\partial f}{\partial x}$ at $(20,4)$. $\dfrac{\partial f}{\partial x} \approx \dfrac{\Delta f}{\Delta x} = \dfrac{81-91}{30-10} = \dfrac{-10}{20} = -\dfrac{1}{2}$

Estimate $\dfrac{\partial f}{\partial y}$ at $(20,4)$. $\dfrac{\Delta f}{\Delta y} = \dfrac{92-80}{6-2} = \dfrac{12}{4} = 3$

Use these partial derivatives to estimate $f(22,4)$.

$$f(22,4) = f(20,4) + \underbrace{2}_{\Delta x} \cdot \underbrace{\left(-\tfrac{1}{2}\right)}_{f_x} = 85 - 1 = 84$$

Use these partial derivatives to estimate $f(20,5)$.

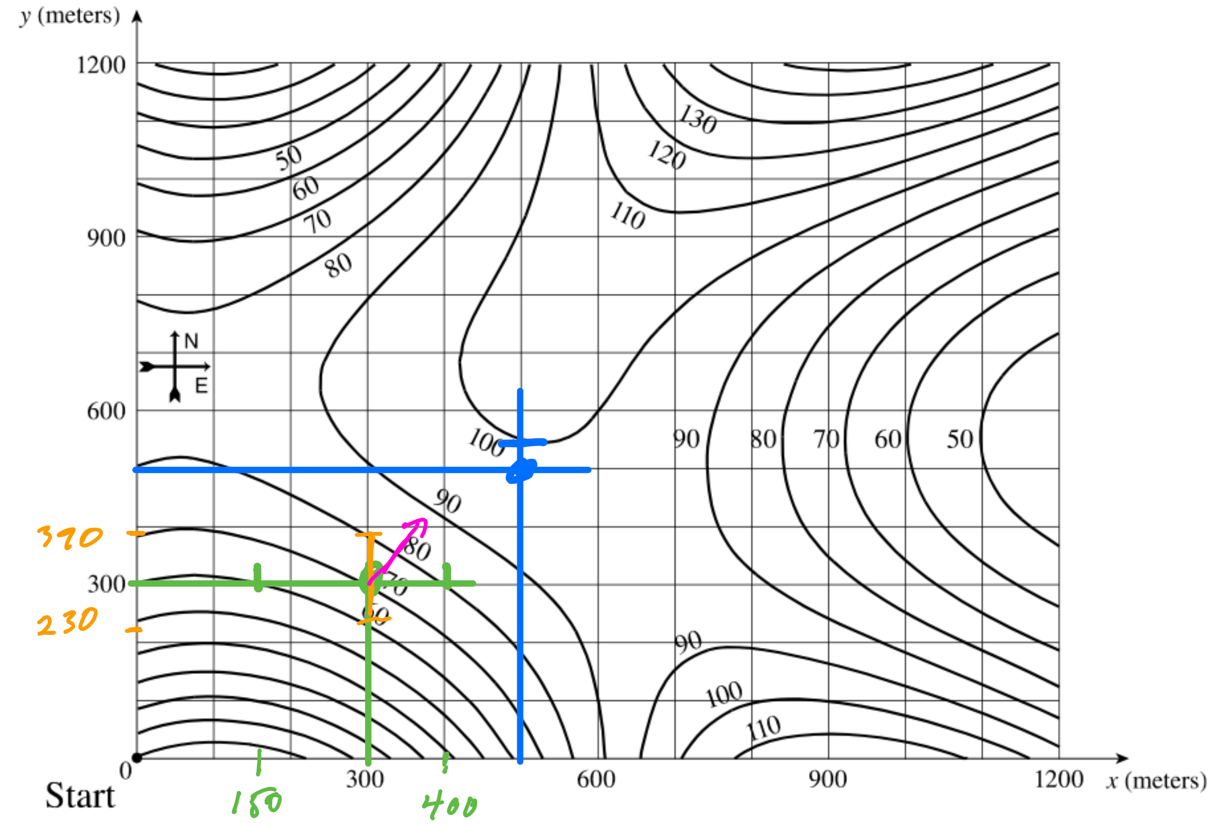
$$f(20,5) = f(20,4) + \underbrace{1}_{\Delta y} \cdot \underbrace{3}_{f_y} = 85 + 3 = 88$$

Estimate $f(22,5)$.

$$f(22,5) = f(20,4) + \Delta x f_x + \Delta y f_y$$
$$= 85 - 1 + 3 = 87$$

Math 213 - Graphical Data

The following is a map with curves of the same elevation of a region in Orangerock National Park.



We define the altitude function $A\,(x,y)$ as the altitude at a point x meters east and y meters north of the origin ("Start").

1. Estimate $A\,(300,300)$ and $A\,(500,500)$.

   70   ~97

2. Estimate $A_x\,(300,300)$ and $A_y\,(300,300)$.

$$A_x \approx \frac{\Delta A}{\Delta x} = \frac{80-60}{400-150} = \frac{20}{250} \simeq 0.08$$

at $(x,y)=(300,300)$

$$A_y \approx \frac{\Delta A}{\Delta y} = \frac{80-60}{390-230} = \frac{20}{160} = 0.13$$

3. What do A_x and A_y represent in physical terms?

slope of a path as we move East

slope of a path as we move North

4. In which direction does the altitude increase most rapidly at the point (300, 300)?

See ↗

5. Use your estimates of A_x (300,300) and A_y (300,300) to approximate the altitude at (320, 310).

$$A(320,310) = A(300,300) + 20A_x + 10A_y$$
$$= 70 + 20 \cdot .08 + 10 \cdot 0.13$$
$$= 70 + 1.6 + 1.3$$
$$= 72.9$$

Math 213 - **More Graphical Data**

1. Refer to the following contour graph.

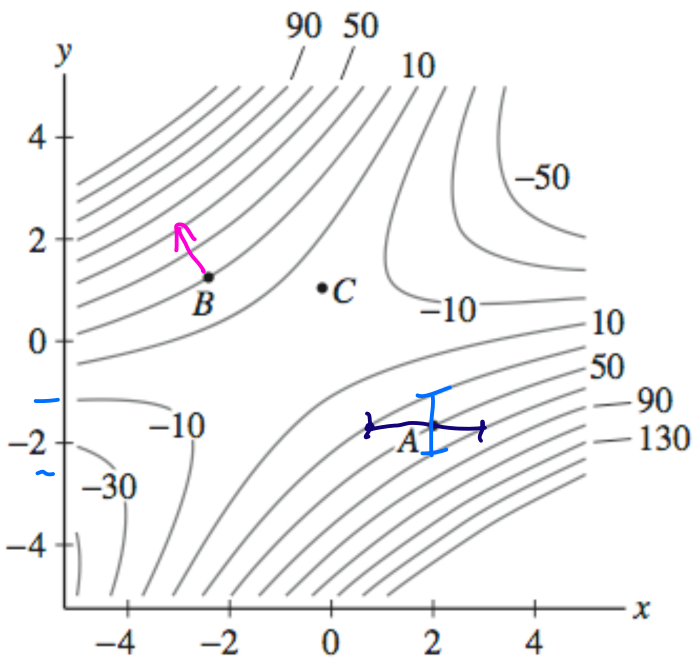


FIGURE 8 Contour map of $f(x, y)$.

a) Estimate f_x and f_y at the point A.

$$f_x \simeq \frac{70-30}{2.8-0.3} = \frac{40}{2.5} = \frac{160}{10} = 16$$

$$f_y = \frac{30-70}{-1-(-3)} = \frac{-40}{2} = -20$$

b) Starting at point B, in which direction does f increase most rapidly?

c) At which of A, B, or C is f_y smallest?

$f_y|^A \simeq -20$ $\quad f_y|^B > 0$ $\quad f_y|^C \simeq 0$

so $f_y|^A$ is smallest

2. Refer to the following contour graph of $f(x,y)$.

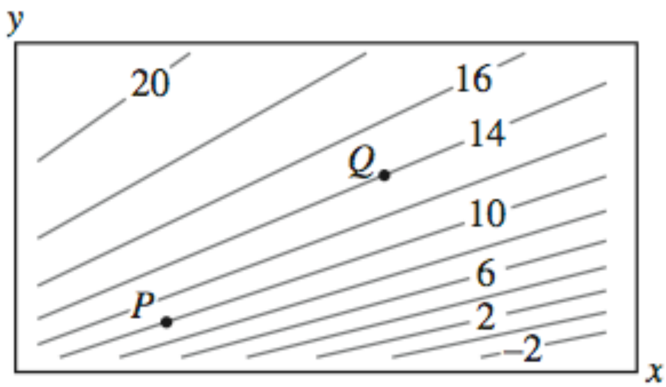


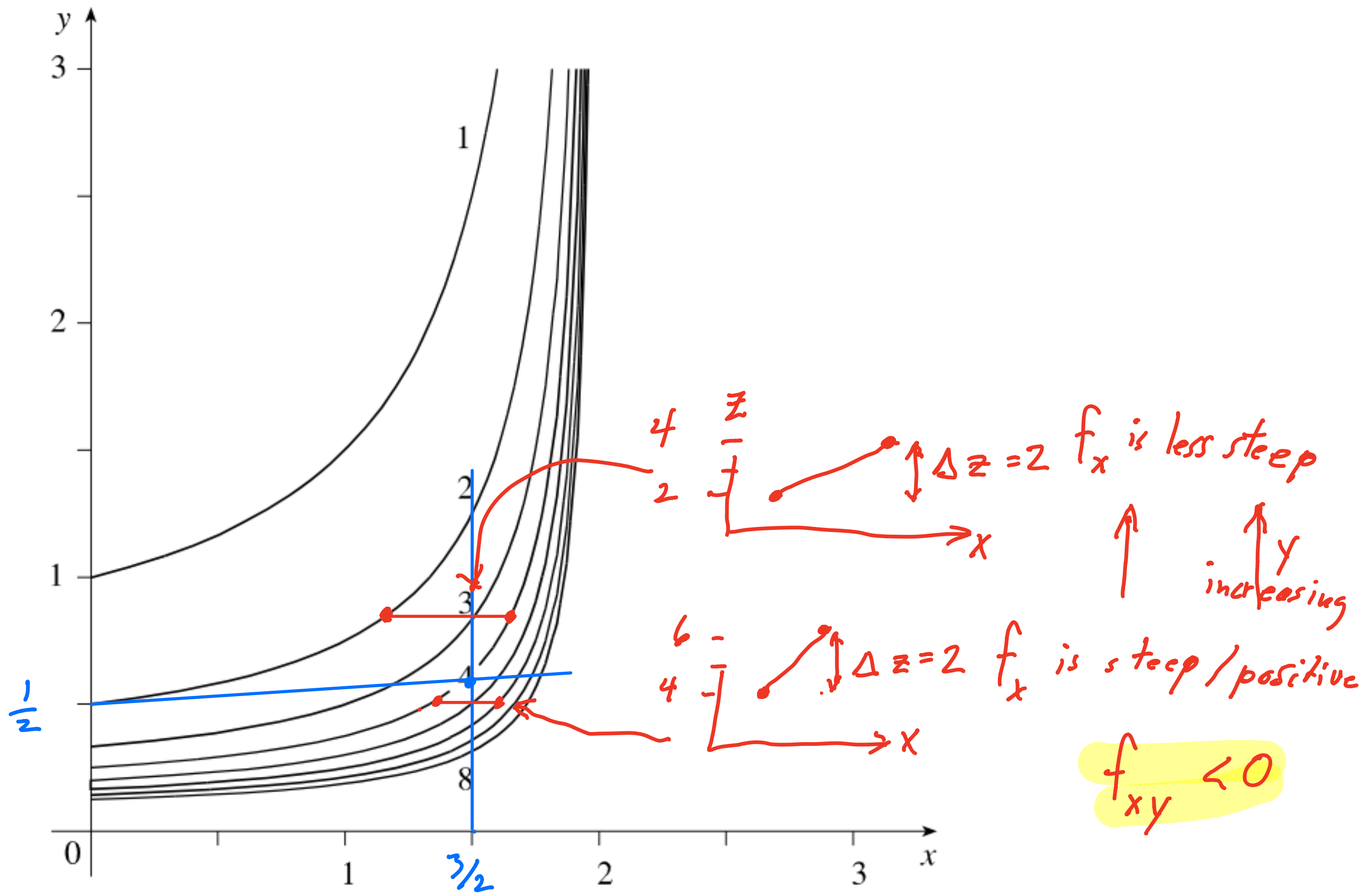
FIGURE 9 Contour interval 2.

a) Explain why f_x and f_y are both larger at P than at Q.

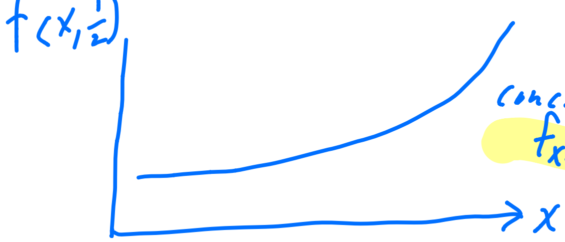
b) Explain why $f_x(x,y)$ is an increasing function of y. That is, for any x, $f_x(x,b_1) > f_x(x,b_2)$ whenever $b_1 > b_2$.

Math 213 - Mixed Partials

The level curves of a function $z = f(x, y)$ are given below.



Use the level curves of the function to decide the signs (positive, negative, or zero) of the derivatives $f_{xx}, f_{yy}, f_{xy}, f_{yx}$, of the function at the point $\left(\frac{3}{2}, \frac{1}{2}\right)$



concave upwards, $f_{xx} > 0$

concave upwards $f_{yy} > 0$