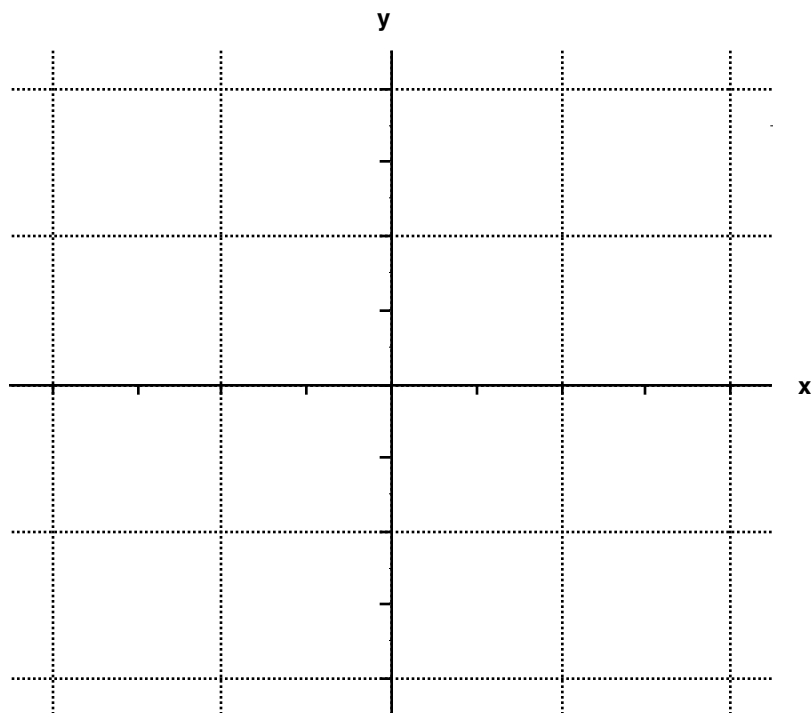


### Math 213 Class 13: Divergence and Curl Homework

1. Let  $\mathbf{F}(x,y) = y \mathbf{i}$ . Carefully sketch several representative vectors – at least one in each quadrant and at least one on each axis.

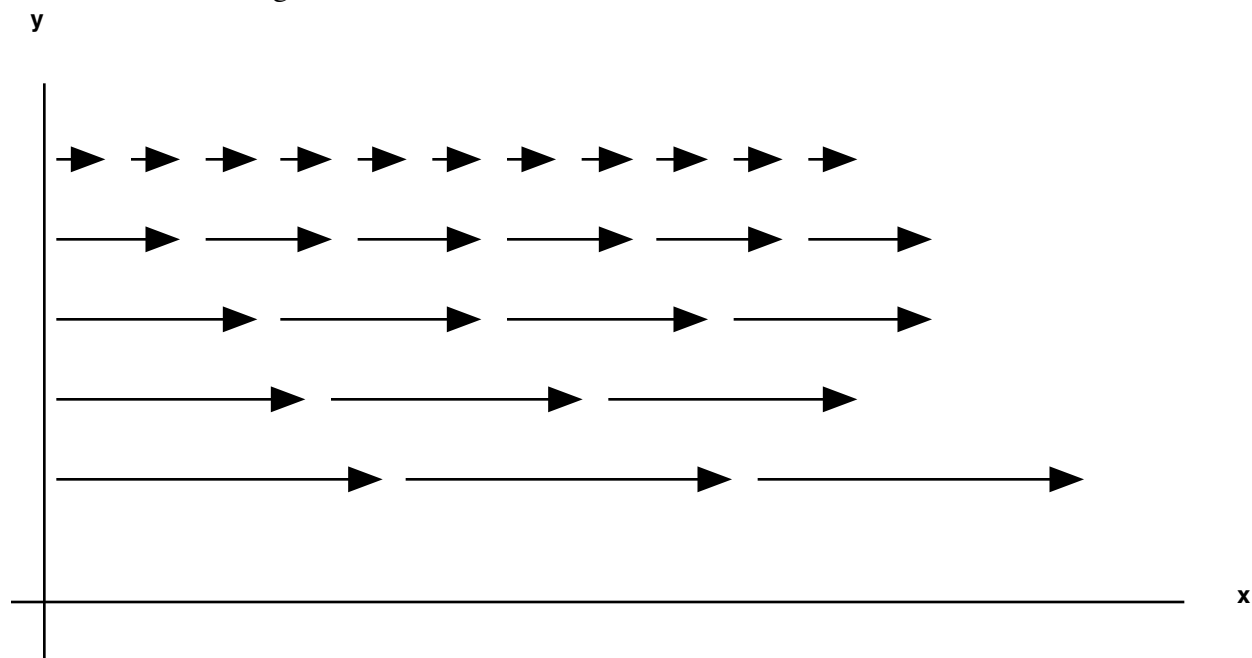


2. Let  $\mathbf{F}(x,y,z) = z^2x \mathbf{i} + y^2z \mathbf{j} - z^2y \mathbf{k}$
- a. Compute the divergence of  $\mathbf{F}$ .

- b. Compute the curl of  $\mathbf{F}$ .

### Math 213 Class 13: Divergence and Curl Homework

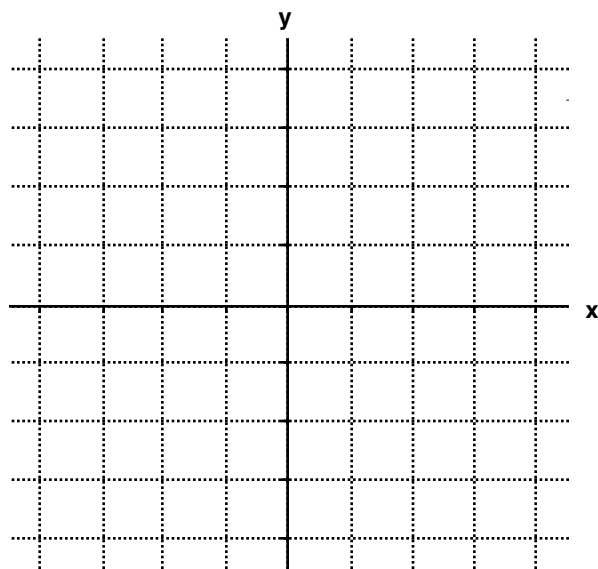
3. The following is an illustration of a two dimensional vector field  $\mathbf{F}$ .



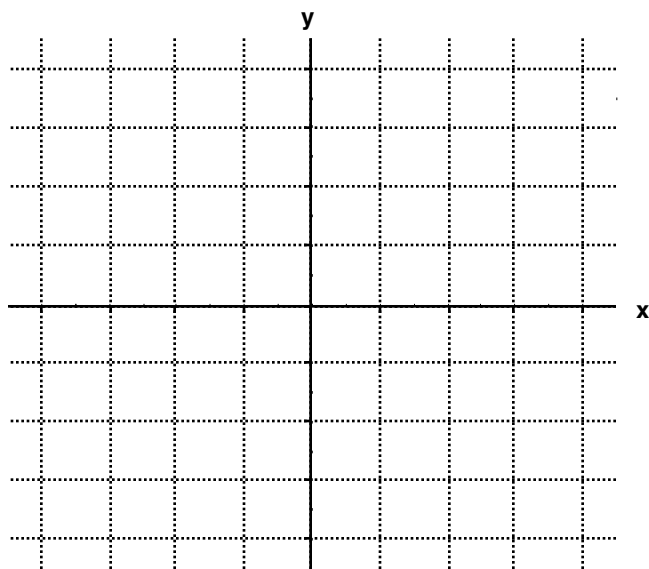
- a. What can you say about the curl of  $\mathbf{F}$ ?
- b. What can you say about the divergence of  $\mathbf{F}$ ?
4. The field  $\mathbf{F}(x,y) = e^y \mathbf{i} + (xe^y + y) \mathbf{j}$  is conservative. Find the potential function of  $\mathbf{F}$ .

**Math 213 Class 13: Vector Fields:** Sketch each of the following vector fields.

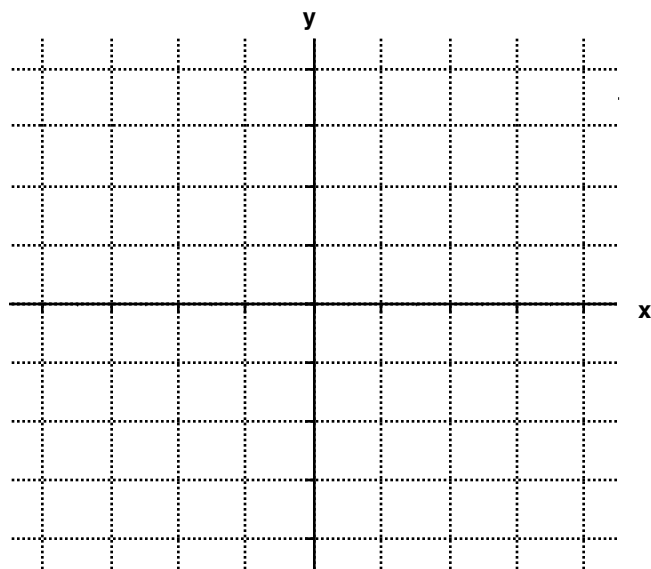
$$\mathbf{F}(x,y) = xy \mathbf{i} + 0 \mathbf{j}$$



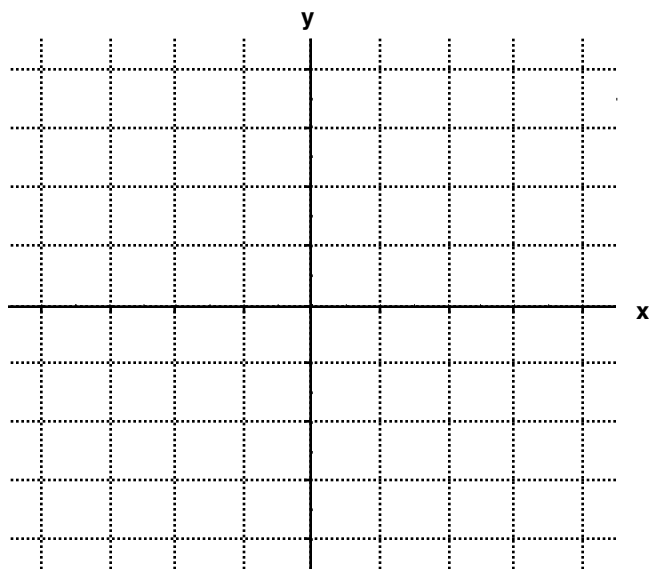
$$\mathbf{F}(x,y) = y \mathbf{i} - x \mathbf{j}$$



$$\mathbf{F}(x,y) = x \mathbf{i} + y \mathbf{j}$$

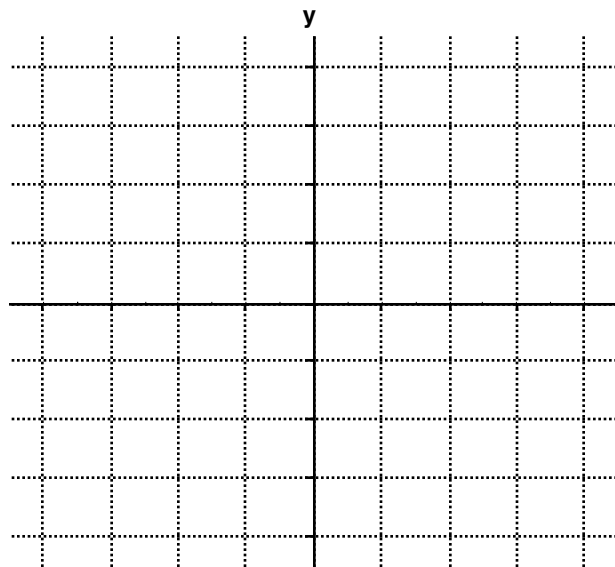


$$\mathbf{F}(x,y) = 2 \mathbf{i}$$

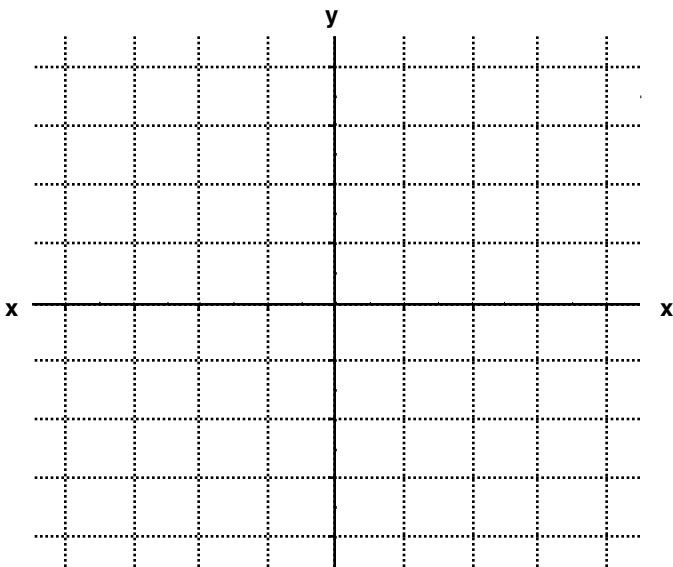


**Math 213 Class 13: Vector Fields:** Sketch each of the following vector fields.

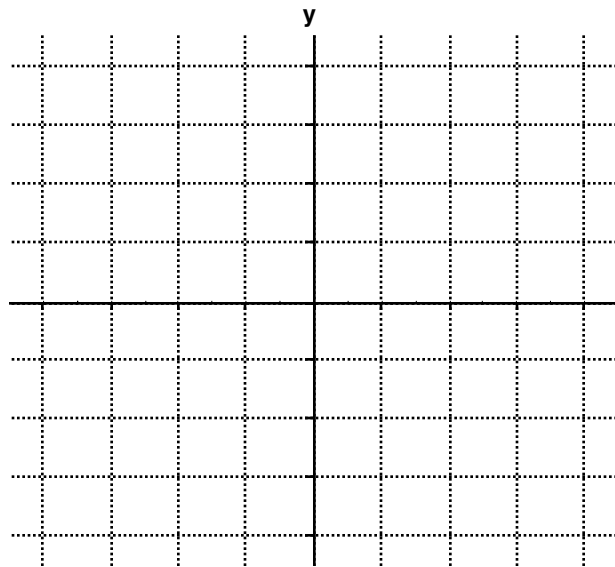
$$\mathbf{F}(x,y) = 2\mathbf{i} + 3\mathbf{j}$$



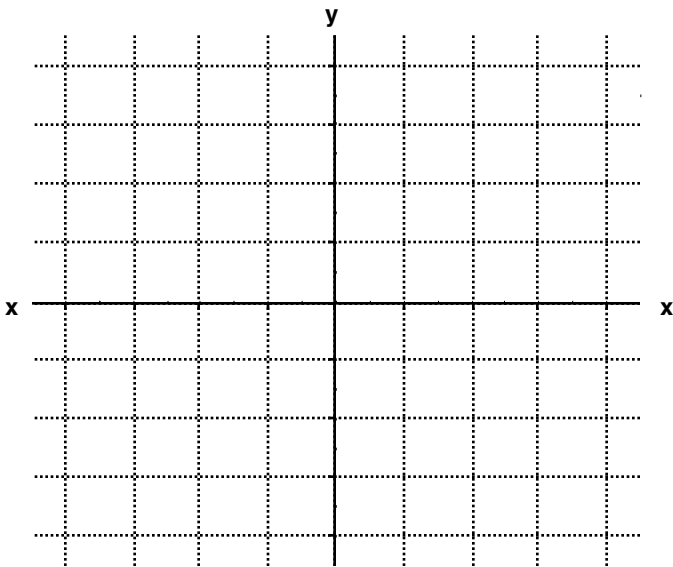
$$\mathbf{F}(x,y) = x\mathbf{i}$$



$$\mathbf{F}(x,y) = 2\mathbf{i} + x\mathbf{j}$$



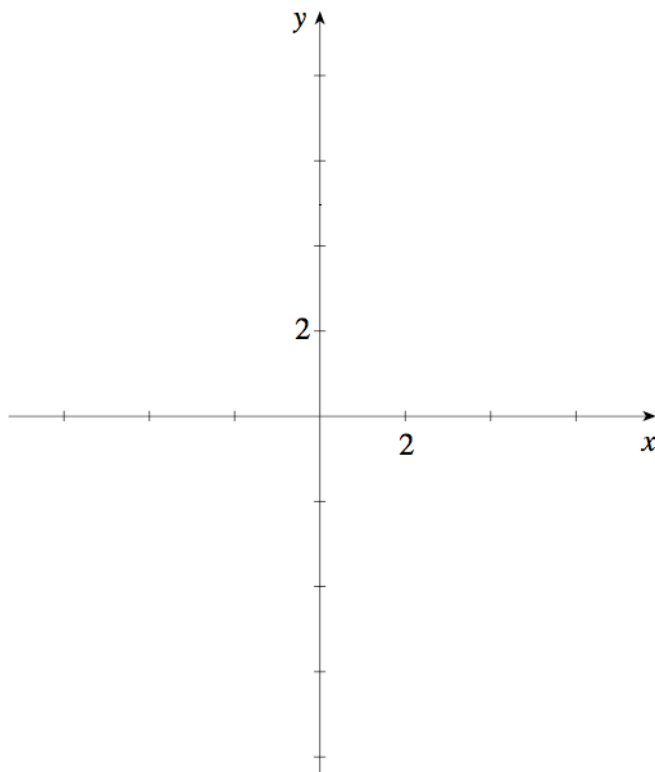
$$\mathbf{F}(x,y) = -x\mathbf{j}$$



### Math 213 Class 13: Gradient Fields and Level Curves

1. Let  $f(x,y) = \frac{x^2}{4} + \frac{y^2}{9}$ .

Draw level curves  $f(x,y) = k$  for  $k = 1, 2, 4$  on the axes below. Then compute the gradient vector field and sketch it at one or two points on each level curve.



## Math 213 Class 13: Gradient Fields and Level Curves

### Finding the Gradient Fields

The following vector fields are conservative. For each one, find a function  $f(x, y)$  or  $f(x, y, z)$  for which it is a gradient field.

1.  $\mathbf{F}(x, y) = 3xy^2 \mathbf{i} + 3x^2y \mathbf{j}$

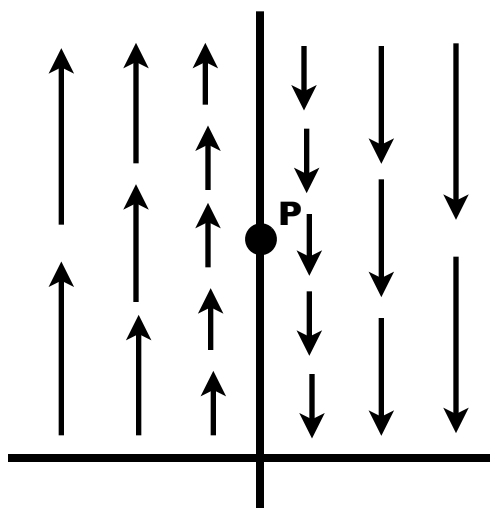
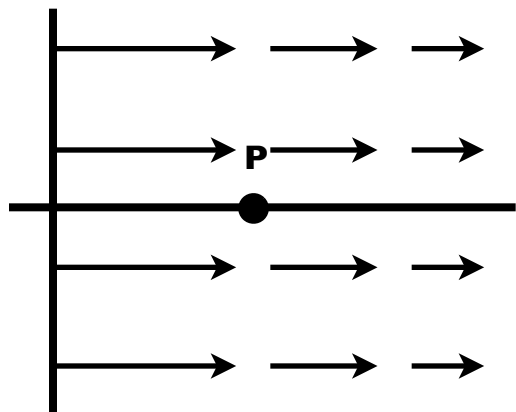
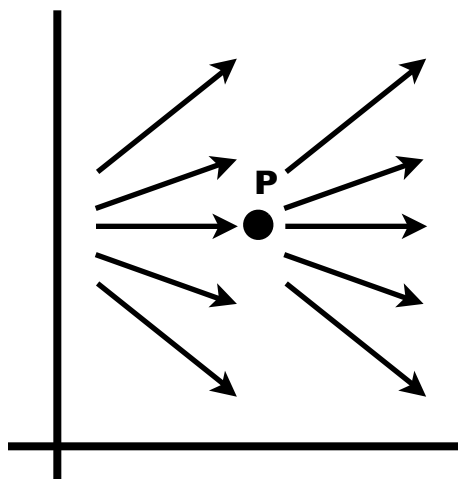
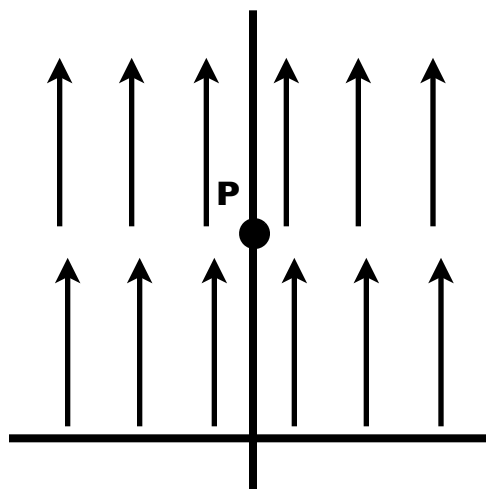
2.  $\mathbf{F}(x, y) = y \sin(xy) \mathbf{i} + x \sin(xy) \mathbf{j}$

3.  $\mathbf{F}(x, y) = (2x + y) \mathbf{i} + (x + 3y^2) \mathbf{j}$

4.  $\mathbf{F}(x, y, z) = yze^{xyz} \mathbf{i} + xze^{xyz} \mathbf{j} + xye^{xyz} \mathbf{k}$

### Math 213 Class 13: Visual Vector Fields

Describe the divergence and the curl of the following vector fields at the point indicated by the black circle.

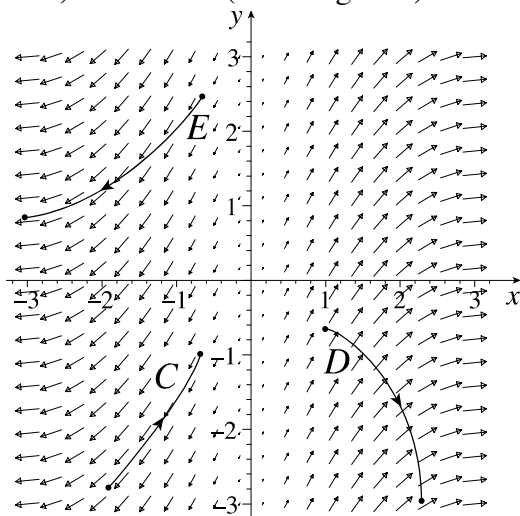


# Math 213 Calculus

## Reading the Text

Read Section 13.2-13.4 and answer the following questions

1. Place the three line integrals  $\int_C \mathbf{F} \cdot d\mathbf{r}$ ,  $\int_D \mathbf{F} \cdot d\mathbf{r}$ ,  $\int_E \mathbf{F} \cdot d\mathbf{r}$  in order from largest (most positive) to smallest (most negative).



2. Evaluate  $\int_C (x + y^2) ds$  where  $C$  is the line segment from  $(0,0)$  to  $(3,0)$ .
3. Is every integral of  $\mathbf{F}(x,y) = (x - y)\mathbf{i} + (x - 2)\mathbf{j}$  independent of path? Why or why not?