Functions and surfaces [9.6]

Dome atop the German Reichstag in Berlin

Functions of more than one variable.

$f(x, y)$

A **function f of two variables** is a **rule** that assigns to each ordered pair of real numbers (x, y) in a set D a *unique* real number denoted by $f(x, y)$.

- The set D is the **domain** of the function f.
- The **range** of f is the set of values that f takes on, that is, the set: $\{f(x, y) | (x, y) \in D\}$.

Often we write such a relation in terms of a dependent variable $z = f(x, y)$ which is a function of the two independent variables x and y.

Examples

- Surface temperature, $T(x, y)$, varies with latitude (y) and longitude (x).

- Height, $h(x, y)$, above sea level on Earth's surface varies with latitude and longitude.

Tabular form

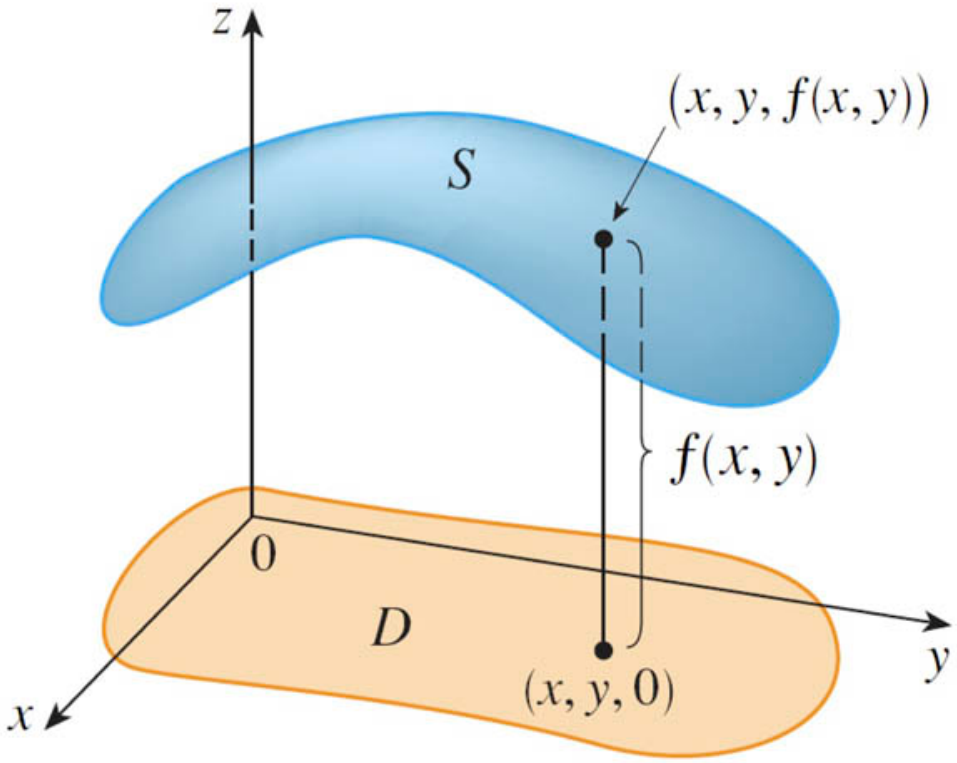
$h(v, t)$: A table of wave height, *h* (in feet) which depends, in the open ocean, on the speed, v, (in knots) of the wind and how long, t, (in hours) it's been blowing.

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

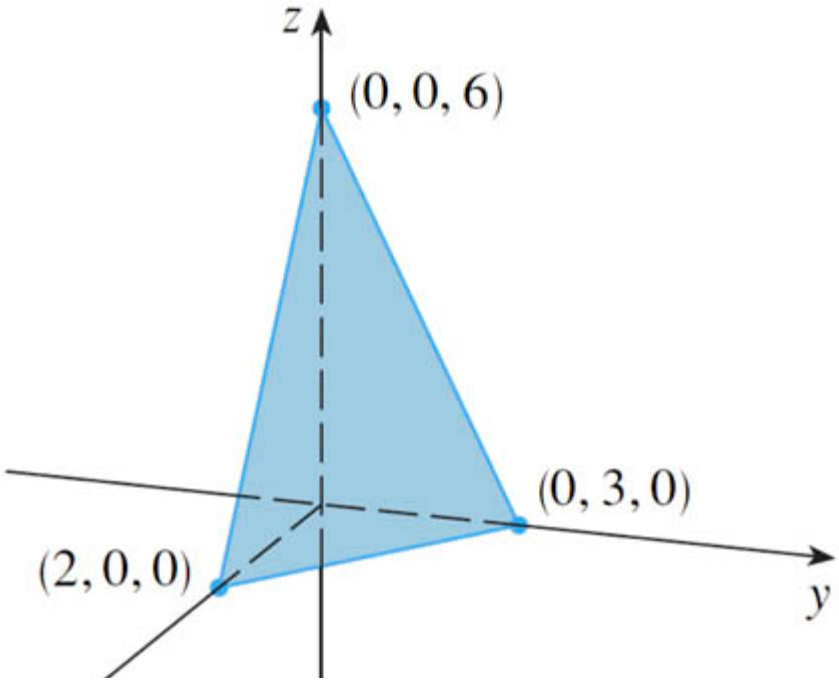
[We had Cyclone Phailin (Oct 2013) and now (May 2019) here comes Cyclone Fani]

Graph



If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

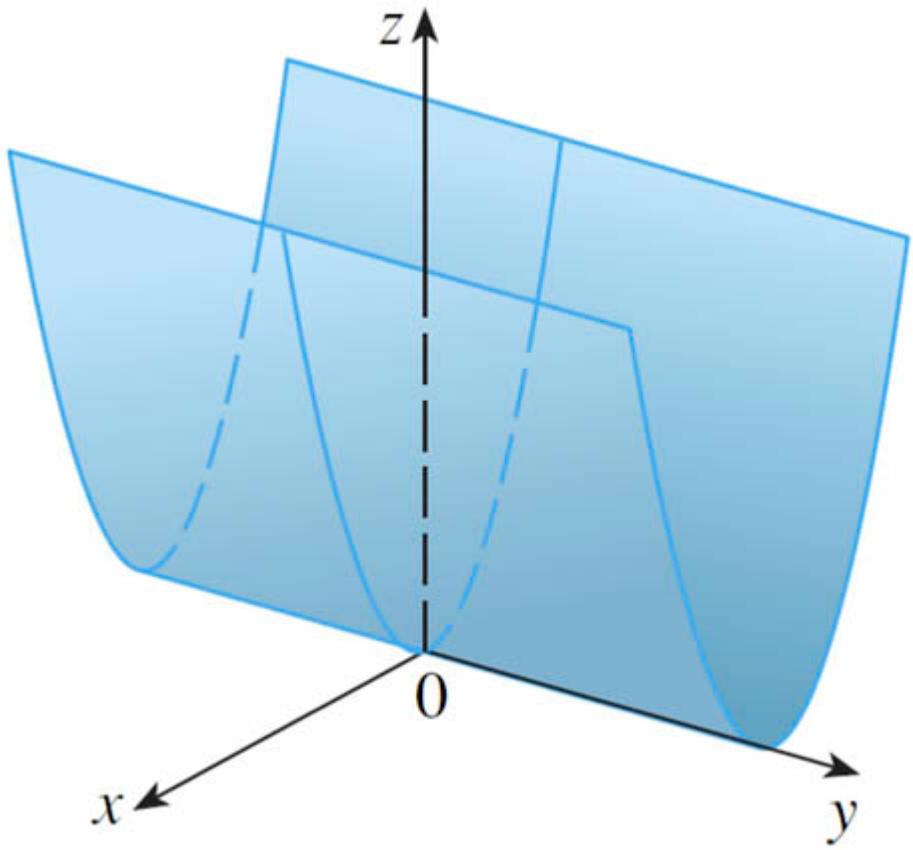
Examples



x ↙ |

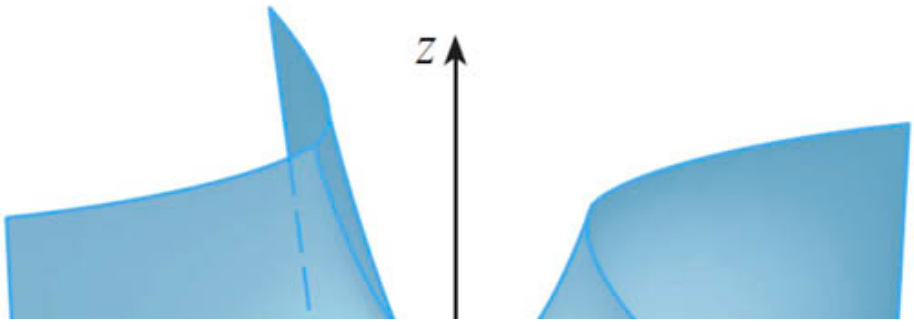
$$z = f(x, y) = 6 - 3x - 2y$$

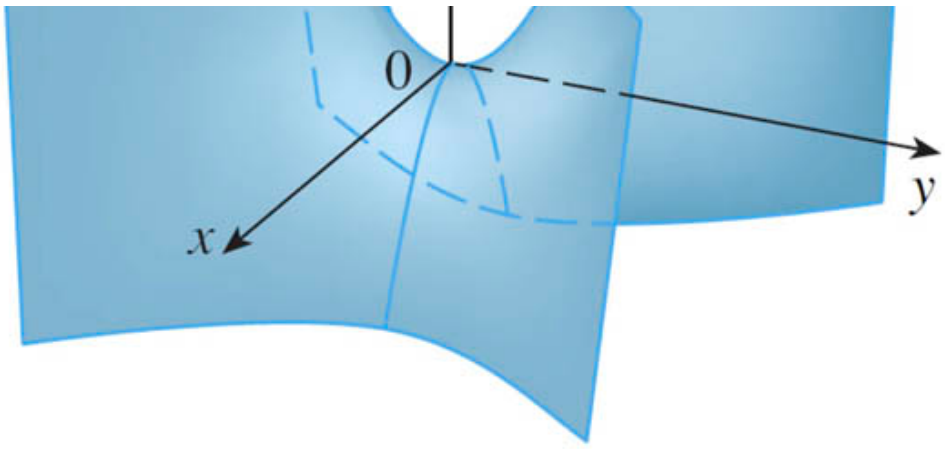
[Verify the intersections with the $x-$, $y-$, and $z-$axes.]



$$z = f(x, y) = x^2$$

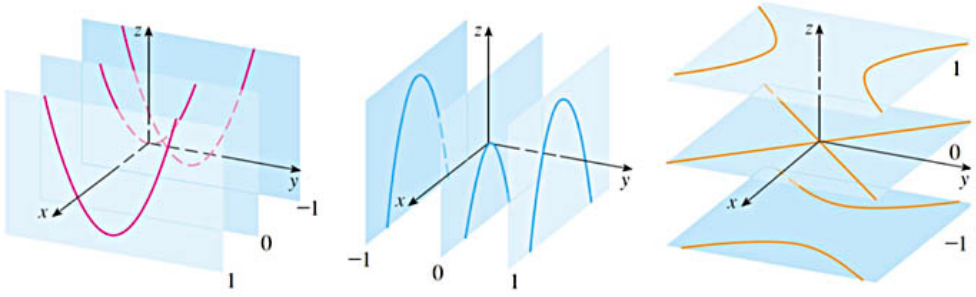
Traces

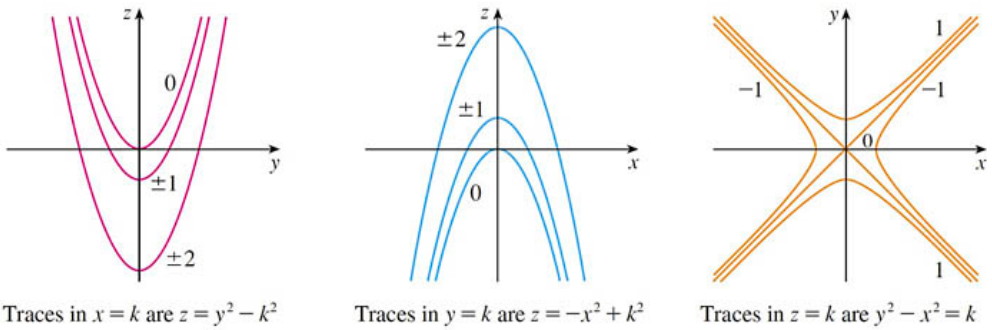




$$z = f(x, y) = y^2 - x^2$$

It is helpful to visualize surfaces by considering intersections of the surface with various planes:



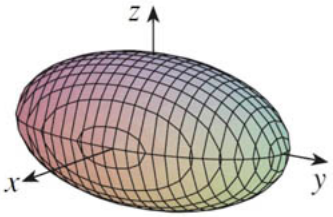


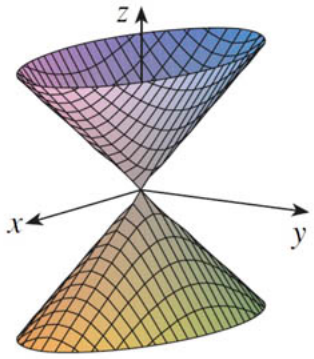
- **Vertical traces** are intersections with some *vertical* plane like $x = k$ or $y = k$.
- **Horizontal traces** are intersections with a *horizontal* plane like $z = k$. (The contours on a contour map are usually horizontal traces of the height above sea level.)

To do

- *Staying Cool*

Quadric surfaces

Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.

Surface	Equation
Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.

To do

- Matching Game - general functions
- Matching Game - quadric functions

Image credits

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