Math 213 Class 05: Unit Tangents and Unit Normals

1. Let $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 2t\,\mathbf{k}$.

   (a) Calculate the unit tangent vector $\mathbf{T}$.

   (b) Calculate the principle unit normal vector $\mathbf{N}$.

2. Curvature

   (a) What is the curvature of a straight line?

   (b) What is the curvature of a circle of radius 4?

3. If a particle moves at constant speed, what can be said about its acceleration? Consider the case where the particle moves in a straight line and the case where the particle moves along a curve. Talk about the normal and tangential components of acceleration.

4. Given the following position function $\mathbf{r}(t) = \left(2 + 3t + 3t^2\right)\mathbf{i} + \left(4t + 4t^2\right)\mathbf{j} + \left(-6\cos t\right)\mathbf{k}$ write $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ at $t = 0$. You do ***not*** need to do really complicated calculations.

5. A particle moves along the curve given by $\mathbf{r}(t) = t\,\mathbf{i} + \frac{\sqrt{6}}{2}t^2\,\mathbf{j} + t^3\,\mathbf{k}$.

a. Find the speed of the particle.

b. Find the tangential component of acceleration.

c. Find the normal component of acceleration.

d. Find the curvature of the curve.

Math 213 Class 05: Lost in Space

1. A space station is located at (0,0,0). A rocket is traveling along the path:
   $\mathbf{r}(t) = (\ln t - 2)\mathbf{i} + \left(\frac{2}{t} + 2\right)\mathbf{j} + (3t - 9)\mathbf{k}$

   If the rocket shuts off its engines at $t = 1$, will the rocket coast into the space station?

2. Planet A is moving on a path: $\mathbf{r}(t) = t^2\,\mathbf{i} + 8\,\mathbf{j} + (5t - 6)\mathbf{k}$
   Planet B is moving along: $\mathbf{s}(t) = (10 - 3t)\mathbf{i} + 2t^2\,\mathbf{j} + t^2\,\mathbf{k}$

   When will the 2 planets collide and what will be the position?

   Which planet will be moving faster when they collide?

   At what angle will they collide?