# Math 213 Calculus III

**Reading the Text**

Read Section 11.2, 11.3, 11.4 and answer the following questions

1. When talking about limits for functions of several variables, why isn't is sufficient to say $\lim_{(x,y)\to(0,0)} f(x,y) = L$ if $f(x,y)$ gets close to $L$ as we approach (0,0) along the $x$ axis and along the $y$ axis. Hint: Consider path independence.
2. Show that $\lim_{(x,y)\to(0,0)} \left(x^2 + y^2\right) = 0$.
3. Consider $\lim_{x,y\to\infty} \dfrac{xy}{3x^2 + 2y^2}$. Why does the limit not exist?
4. Suppose that $f(x,y)$ is continuous everywhere. Assume that $f_x(1,1) = 2.\quad f_y(1,1) = -2,\quad f_{xy}(1,1) = 3$. Can we compute $f_{yx}(1,1)$ from this information alone?
5. Find a function $f(x,y)$ for which $\dfrac{\partial f}{\partial x} = x + y,\quad \dfrac{\partial f}{\partial y} = x$

**Math 213 Class 06: Contour Maps**

Draw a contour map for each of the following functions.

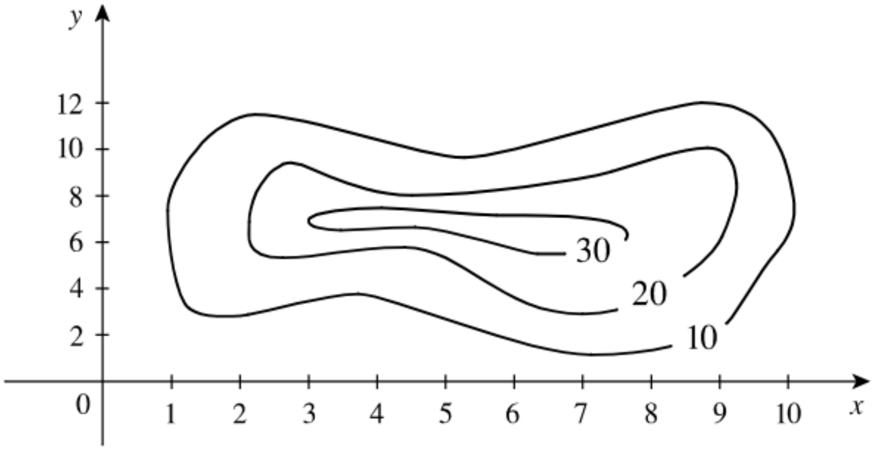
$f(x,y) = x + y + 1$

$f(x,y) = x + 2y - 1$

$f(x,y) = y - x^2$

**Math 213 Class 06: Contour Maps**

Consider the following contour map of a continuous function $f(x,y)$:




1. For approximately what values of $y$ is it true that $10 \le f(5, y) \le 30$?

2. What can you estimate $f(2,4)$ to be and why?

3. Do we have any good estimates for $f(6,6)$? Explain.

4. How many values of $y$ satisfy $f(7, y) = 20$?

5. How many values of $x$ satisfy $f(x, 8) = 20$?

**Math 213 Class 06: Level Surfaces**

It can be difficult to visualize functions of three variables. One way to do it is by thinking of each level surface as representing a different point in time. As we let t vary in the equation for the level surface $f(x, y, z) = t$ we can think of the function $f(x, y, z)$ as a surface whose shape and size varies as time changes.

Consider the function $f(x,y,z) = x^2 + y^2 - z^2$.

1. What is the level surface $f(x,y,z) = 0$?

2. What is the level surface $f(x,y,z) = 1$?

3. For $t > 0$, what do the level surfaces $f(x,y,z) = t$ look like?

4. What is the level surface $f(x,y,z) = -1$?

5. Describe all the level surfaces $f(x,y,z) = t$.