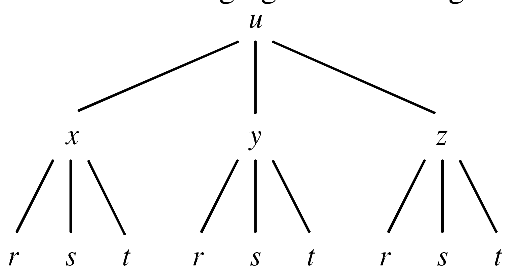
Math 213 Calculus III

Reading the Text

1. Read Section 11.5-11.6 and answer the following questions
   What was the following figure illustrating in the text? Specifically, how was it used?



2. Suppose that $f(x,y) = -x + y^2, \quad x = u^2 + v^3, \quad y = 2u - 3v$. Compute $\dfrac{\partial f}{\partial u}$ when $u = 1, \quad v = -1$.
3. The text shows that $D_{\mathbf{u}} f(x,y) = \nabla f(x,y) \cdot \mathbf{u}$ where **u** is a unit vector. Interpret this in terms of the scalar projection of the gradient vector onto **u**.
4. If $f(x,y) = xy^2 + x$, what is $\nabla f(x,y)$?

Math 213 Class 07: Limits

Try to estimate the following limits by graphing or by plugging small values of x and y into the appropriate functions. Remember that path independence is important - so try different paths. If the limit exists (or if the limit is $\pm\infty$) indicate that. If the limit does not exist, explain why.

1. $\lim_{(x,y)\to(0,0)} 5$

5. $\lim_{(x,y)\to(0,0)} \dfrac{x^3y^2}{x^4 + y^8}$

2. $\lim_{(x,y)\to(0,0)} \dfrac{e^{xy}}{x^2 + y^2}$

6. $\lim_{(x,y)\to(0,0)} \dfrac{x^3}{x^2 + y^2}$

3. $\lim_{(x,y)\to(0,0)} \ln\left(x^2 + y^2\right)$

7. $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

4. $\lim_{(x,y)\to(0,0)} \dfrac{\sin\left(2\left(x + y\right)\right)}{x + y}$

8. $\lim_{(x,y)\to(0,0)} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 25} - 5}$

Math 213 Class 07: Tabular Data

Wave Heights on the Open Sea

The wave heights h in the open sea depend on the speed v of the wind (knots) and the length of time t that the wind has been blowing at that speed (hours). Values for the function $h = f(v,t)$ are in the following table.

v\t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

Questions:

1. What is the value of $f(40,15)$? What is its meaning?

2. What is the meanings of the function $h = f(30,t)$? $h = f(v,30)$?

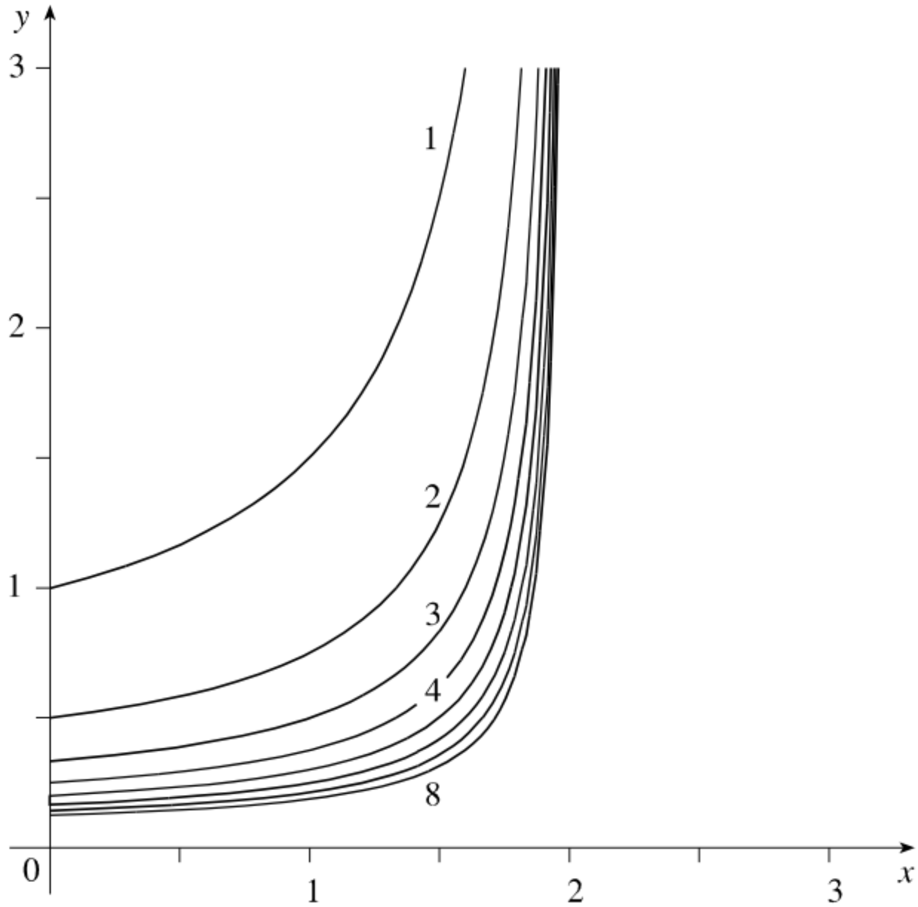
3. Estimate the values of $\frac{\partial f}{\partial v}(40,20)$ and $\frac{\partial f}{\partial t}(40,20)$ and interpret their meanings.

4. Find a linear approximation to the wave height function when v is near 40 knots and t is near 20 hours. (Round the numerical coefficients to two decimal places).

5. Using the linear approximation, estimate the wave heights when the wind has been blowing for 24 hours at 43 knots. (Round the answer to two decimal places).

Math 213 Class 07: Mixed Partials

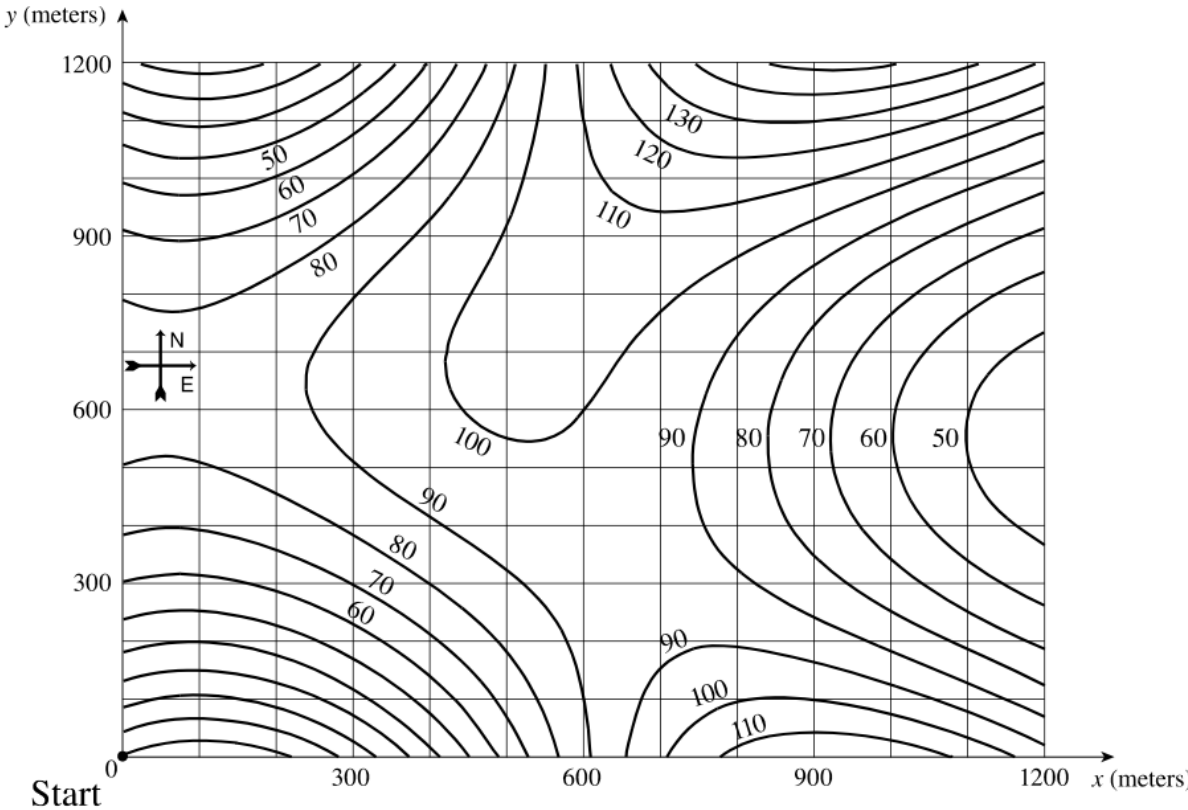
The level curves of a function $z = f(x, y)$ are given below.



Use the level curves of the function to decide the signs (positive, negative, or zero) of the derivatives $f_{xx}, f_{yy}, f_{xy}, f_{yx}$, of the function at the point $\left(\frac{3}{2}, \frac{1}{2}\right)$

Math 213 Class 07: Graphical Data

The following is a map with curves of the same elevation of a region in Orangerock National Park.



We define the altitude function $A(x,y)$ as the altitude at a point x meters east and y meters north of the origin ("Start").

1. Estimate $A(300,300)$ and $A(500,500)$.

2. Estimate $A_x(300,300)$ and $A_y(300,300)$.

3. What do A_x and A_y represent in physical terms?

Math 213 Class 07: Graphical Data

4. In which direction does the altitude increase most rapidly at the point $(300, 300)$?

5. Use your estimates of A_x (300,300) and A_y (300,300) to approximate the altitude at (320, 310).