Math 213 Calculus III

Reading the Text

Read Section 11.7-11.8 and answer the following questions

1. Can a differentiable function f have a local maximum at a point (a,b) with $f_x(a,b) = 3$?
2. Give an example of a function f with the property that $f_x(a,b) = 0$, $f_y(a,b) = 0$ and f does *not* have a local maximum or minimum at (a,b).
3. How does the equation $\nabla f(x,y) = \lambda \nabla g(x,y)$ subject to the constraint $g(x,y) = k$ lead to three equations with three unknowns? What are the unknowns?

Math 213 Class 08: Heat Seeking Kitten

Let $T(x,y) = x^2 - 2xy$ be the temperature at a point (x,y) in the region bounded by the curves $y = x$ and $y = x^2$. Suppose that a kitten is crawling around the region.

a. At $\left(\frac{1}{2},\frac{1}{3}\right)$, in which direction should the cat go to cool down as quickly as possible?

b. At $\left(\frac{1}{2},\frac{1}{3}\right)$, in what direction(s) should the kitten go to maintain its current temperature?

c. Where is the hottest point in the region? Explain your answer.

d. If, at $\left(\frac{1}{2},\frac{1}{3}\right)$, the cat moves in such a way that for each change in its x direction of 2 units, the change in the y direction is -1 unit, find $\frac{dT}{dt}$, the change in the temperature, from the kitten's point of view.

Math 213 Class 08: Gradient and Directional Derivative

1. Suppose you are at a the point with coordinates $(0.6, 0.8)$ in a region where the altitude is given by $f(x,y) = \sin(\pi x + 2\pi y)$. In what direction(s) should you go in order to stay at the same elevation? Justify your answer with a brief description of how you solved the problem.

2. When is the directional derivative of a function equal to zero?

3. Suppose that you are given only the following information about a function f:

$$f(8,5) = 33.1$$
$$f(8.01,5) = 33.3$$
$$f(8,5.02) = 33.0$$

Estimate

$$f_x(8,5)$$
$$f_y(8,5)$$
$$\nabla f(8,5)$$
$$D_{\mathbf{u}} f(8,5) \qquad \text{where } \mathbf{u} = \frac{1}{5}(3\,\mathbf{i} + 4\,\mathbf{j})$$

Math 213 Class 08: Gradient and Directional Derivative

4. Find the path of a heat seeking particle placed at the point P(10,10) on a metal plate with a temperature field given by $T(x,y) = 400 - 2x^2 - y^2$: Print out a contour map (that you make with Mathematica) of the region near P(10,10) and sketch a plausible path on the contour map.

5. The temperature field in the neighborhood of $\left(\frac{\pi}{4},0\right)$ is given by $T(x,y) = \sqrt{2}e^{-y}\cos(x)$.

   Find the path followed by a heat seeking particle originating at $\left(\frac{\pi}{4},0\right)$. (As above, print out a contour map)