Math 213 Calculus III

Reading the Text
Read Section 12.4 and 12.6 (we are skipping 12.5) and answer the following questions

1. Is the area of a polar region $R = \left\{(r,\theta) \middle| \ a \le r \le b, \quad \alpha \le \theta \le \beta\right\}$ equal to $(b-a)\cdot(\beta-\alpha)$?
2. Convert $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\left(x^2+y^2\right)dxdy$ to polar coordinates.
3. Write an integral that will give the area of a cross-section of the paraboloid $z = x^2 + y^2$ that for $z = 9$.

Math 213 Class 10: Double Integrals Practice

1. $\int_0^2 \int_0^2 \left(x^2 - y^2\right) dy\, dx$

2. $\int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \cos\left(2x + y\right) dy\, dx$

3. $\int_0^2 \int_1^3 x^3 y\, dy\, dx$

4. $\int_{-1}^1 \int_0^{\pi} x^2 \sin y\, dy\, dx$

5. $\int_0^2 \int_{x^2}^{2x} \left(x^2 + 2y\right) dy\, dx$

Math 213 Class 10: Double Integrals Practice

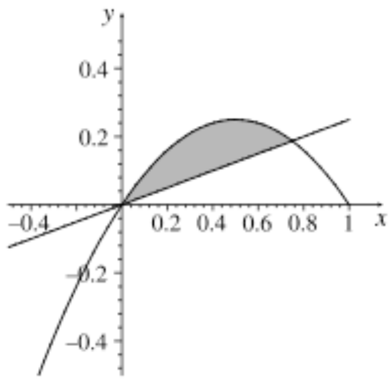
6. $\int_0^3 \int_0^{\sqrt{9-x^2}} 4x\,dy\,dx$

7. $\int_0^2 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx$

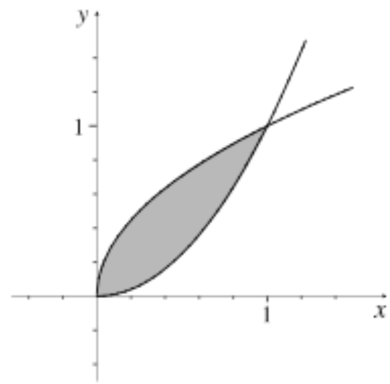
Math 213 Class 10: Double Integrals and Areas

1. Write double integrals that represent the following areas.

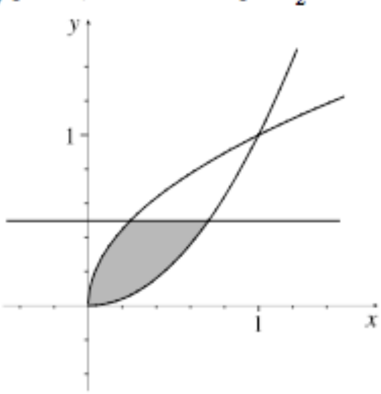
(a) The area enclosed by the curve $y = x - x^2$ and the line $y = \dfrac{x}{4}$



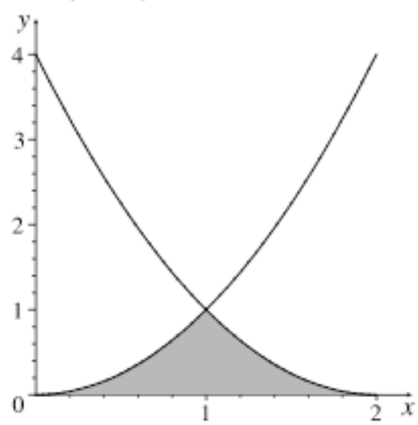
(b) The area enclosed by the curves $y = \sqrt[4]{x}$ and $\sqrt{y} = x$



(c) The area enclosed by the curves $y = \sqrt{x}$ and $\sqrt[4]{y} = x$, and the line $y = \frac{1}{2}$



(d) The area enclosed by the curves $y = x^2$ and $y = (x-2)^2$, and the line $y = 0$



2. What solid region of $\mathbb{R}^3$ do you think is represented by $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2}\, dy\, dx$?

Math 213 Class 10: Double Integrals from Data

For the function whose values are given below,

	x=20	30	40	50	60
y=80	77	78	79	81	82
85	82	84	86	88	90
90	87	90	93	96	100
95	93	96	101	107	114
100	99	104	110	120	132

Estimate $\int_{80}^{100}\int_{20}^{60} f(x,y)\,dx\,dy$ using

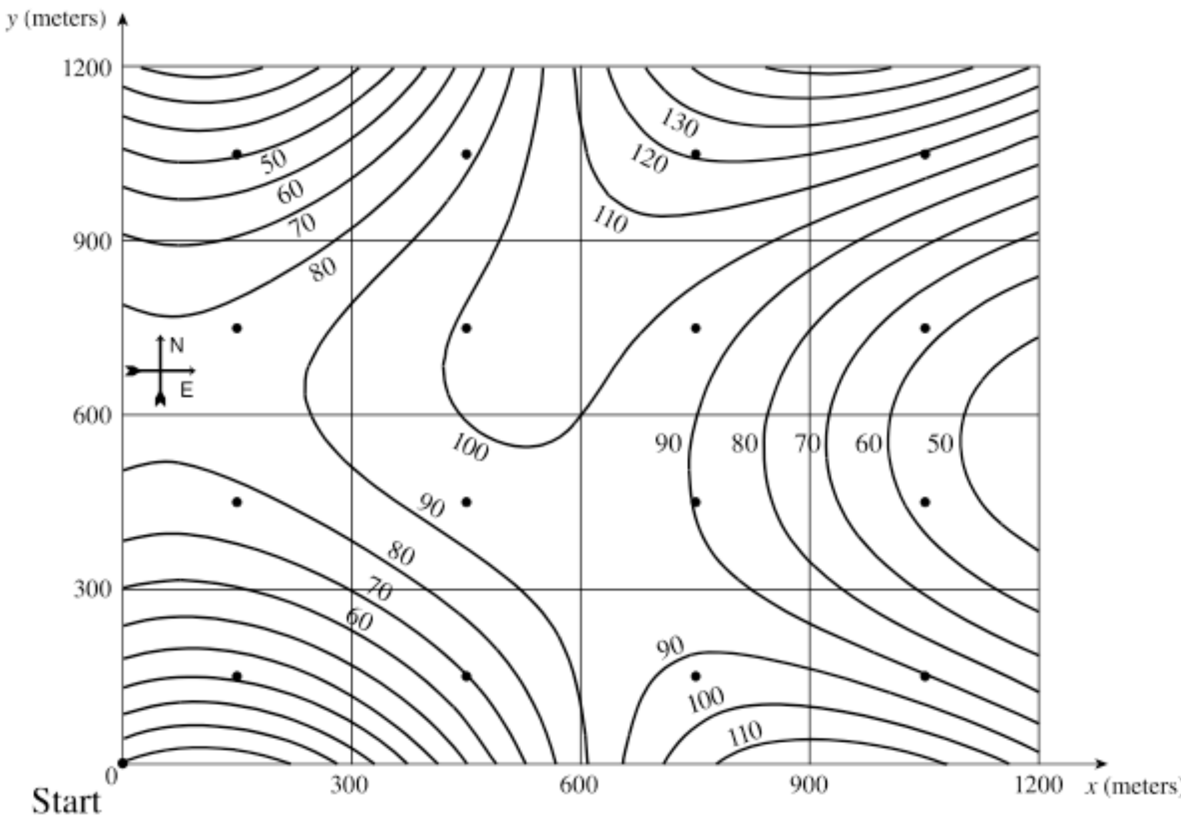
1. midpoints

2. points farthest away from the origin

Math 213 Class 10: Double Integrals Graphically

Back to the Park

The following is a map with curves of the same elevation of a region in Orangerock National Park:



Estimate (numerically) the average elevation over this region using the Midpoint Rule.

Math 213 Class 10: Double Integrals: Problems

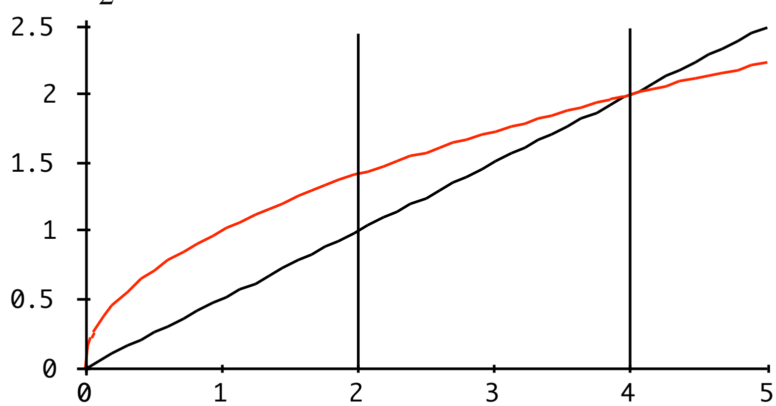
1. Evaluate the integral:

$$\int_0^1 \int_0^{1+x} (3x + 2y)\, dy\, dx$$

.

2. Integrate $\iint_R xy\, dA$ where R is the region bounded by the graphs of

$y = \sqrt{x}, \quad y = \frac{1}{2}x, \quad x = 2, \quad x = 4$ as seen below.



3. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sin\left(\pi x^3\right) dx dy$ by reversing the order of integration.

4. Set up an integral for both orders of integration. Do *not* evaluate.

$$\iint_R \frac{y}{x^2 + y^2} dA$$

where R is the triangle bounded by $y = x, \quad y = 2x, \quad x = 2$.

a. Sketch the region R.

b. Set up the integral for the order: $dy\,dx$.

c. Set up the integral for the order: $dx\,dy$.