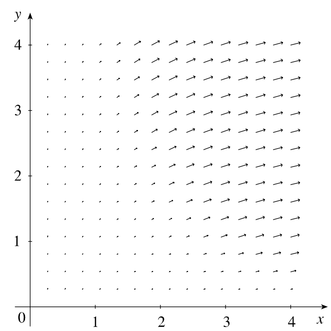
Math 213 Calculus III

Reading the Text

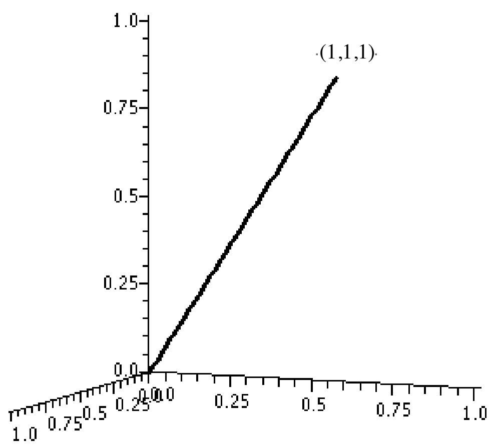
Read Section 13.4, 13.6-13.8 and answer the following questions

1. Why is Green's Theorem useful?
2. If we know that $P(x,y) = 0$ and $Q(x,y) = 0$ on the boundary $C = \partial D$ of a boundary D. then $\int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$?
3. What do we know about the unit normal vector to a closed surface if that surface has positive orientation.
4. Is it possible for a closed oriented curve C to be the boundary of more than one smooth oriented surface?
5. Is it possible for a vector field **F** to have curl **F** = **0** and not be conservative?
6. Is the divergence of the vector field on the right positive, negative or zero at (2,2)?

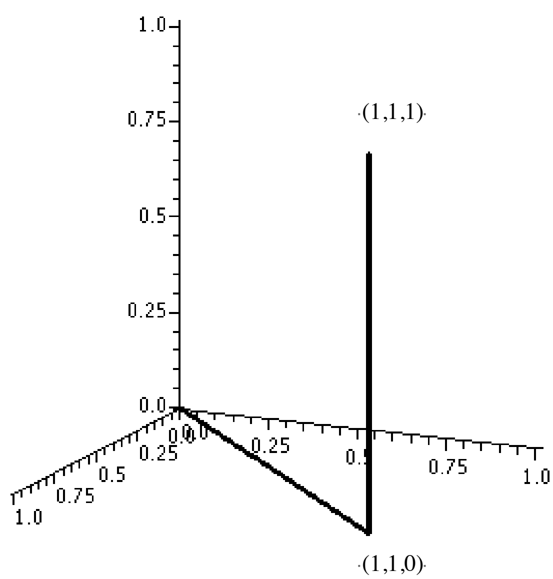


Math 213 Class 14: Scalar Line Integrals

1. Let $f(x,y,z) = x - 3y^2 + z$.
   Integrate f over the line segment C joining the origin (0,0,0) and the point (1,1,1).



2. Let $f(x,y,z) = x - 3y^2 + z$.
   Integrate f over the path shown below joining the origin (0,0,0) and the point (1,1,1).



Math 213 Class 14: Computing Vector Line Integrals

1. Compute the line integral of $\mathbf{F} = x^2\,\mathbf{i} + y^4\,\mathbf{j} + z^6\,\mathbf{k} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$ over the paths $\mathbf{r}_1$: $x = t^3$, $y = t$, $z = t^2, 0 \leq t \leq 1$ and $\mathbf{r}_2$: $x = t$, $y = t^2$, $z = t^3$, $0 \leq t \leq 1$.

2. Compute the line integral of $\mathbf{F} = x^2\,\mathbf{i} + y^4\,\mathbf{j} + z^6\,\mathbf{k}$ over the path $\mathbf{r}_3$: $x = t$, $y = t$, $z = t$, $0 \leq t \leq 1$.

3. Find $g\,(x, y, z)$ so that $\mathbf{F} = \nabla g$.

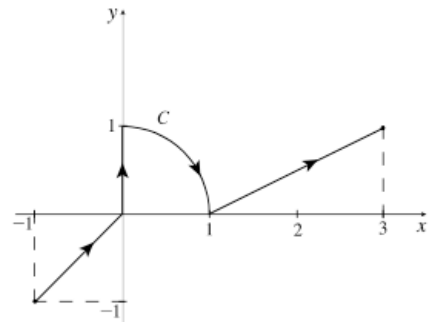
Math 213 Class 14: Computing Vector Line Integrals

4. Compute $\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\mathbf{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\mathbf{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\mathbf{k}$. How could you have arrived at this answer directly by looking at the vector field?

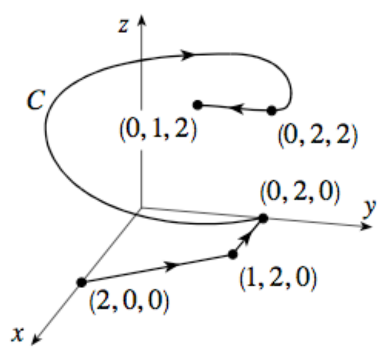
5. Let $\mathbf{r} = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 0\mathbf{k}$, $0 \leq t \leq 2\pi$ be a parametrization of the unit circle. First make a conjecture as to the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$, and then compute it.

Math 213 Class 14: Think before you compute

1. Compute $\int_C (ye^{xy}\,\mathbf{i} + xe^{xy}\,\mathbf{j}) \cdot d\mathbf{r}$ for the curve C shown below.



2. Compute $\int_{\mathcal{C}} \left(yz^2\,\mathbf{i} + xz^2\,\mathbf{j} + 2xyz\,\mathbf{k}\right) \cdot d\mathbf{r}$ for the curve C shown below.



Math 213 Class 14: Line Integrals HW

1. Let C be the line segment from the point (0,0,0) to the point $(1,-3,2)$. Find
$\int_C \left(x + y^2 - 2z\right) ds$

2. Find the work done by the force $\mathbf{F}(x,y) = -x\,\mathbf{i} + 2y\,\mathbf{j}$ on an object moving along the curve C: $y = x^3$, from (0,0) to (2,8).

3. Evaluate $\int_C e^x \sin y \, dx + e^x \cos y \, dy$ where C is the ellipse $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$.

4. Find the work done by the force field $\mathbf{F} = y^2\,\mathbf{i} + x\,\mathbf{j}$ on an object moving in a straight line segment from $(-5,3)$ to $(0,2)$.

5. Let C be the line segment from the point $(0,0,0)$ to the point $(-1,4,3)$. Find $\int_C (x - 3y^2 + z)\,ds$.

6. Let $f(x,y,z) = 8x^2 + xy + z^2$.

a. Find a vector function $\mathbf{F}$ that has potential function $f(x,y,z)$.

b. Evaluate $\int_C \mathbf{F} \bullet d\,\mathbf{R}$ where $\mathbf{R}(t) = \cos^4 t\,\mathbf{i} + 5\sin^7 t\,\mathbf{j} + \mathbf{k}$ and $0 \le t \le \frac{\pi}{2}$

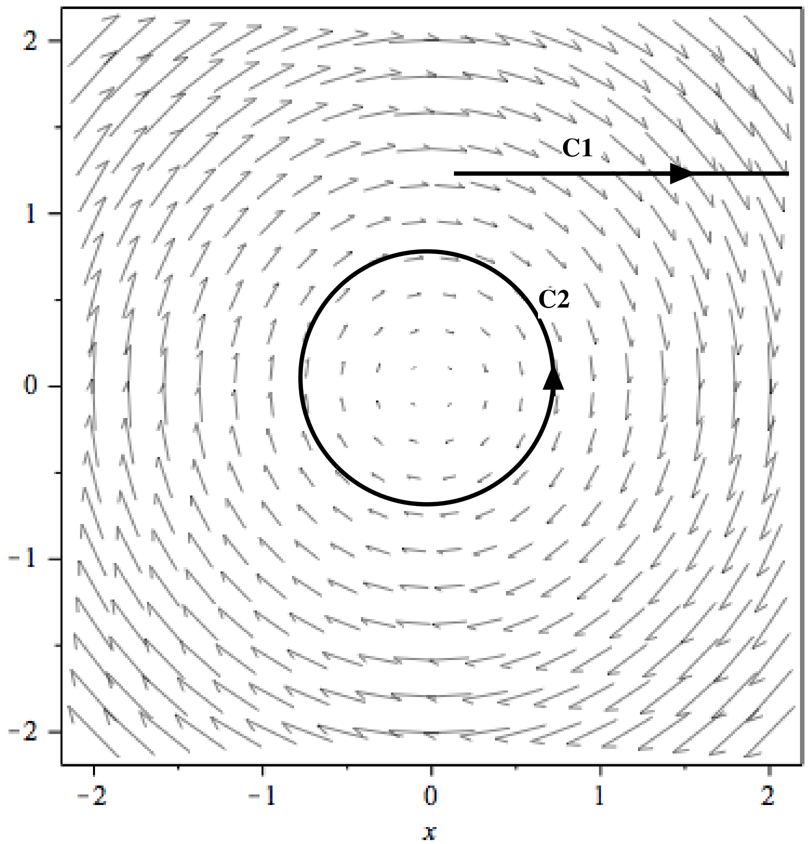
7. Let $\mathbf{F}(x,y) = (2x+y)\cos(x^2+xy)\,\mathbf{i} + \left(x\cos(x^2+xy)+1\right)\mathbf{j}$

a. Show that $\mathbf{F}$ is a conservative vector field.

b. Let C be the curve parametrized by $\mathbf{r}(t) = \sin t\,\mathbf{i} + (1-\cos t)\,\mathbf{j}$ with $0 \le t \le \pi$.

Find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

8. Let **F** be the vector field given in the diagram below.

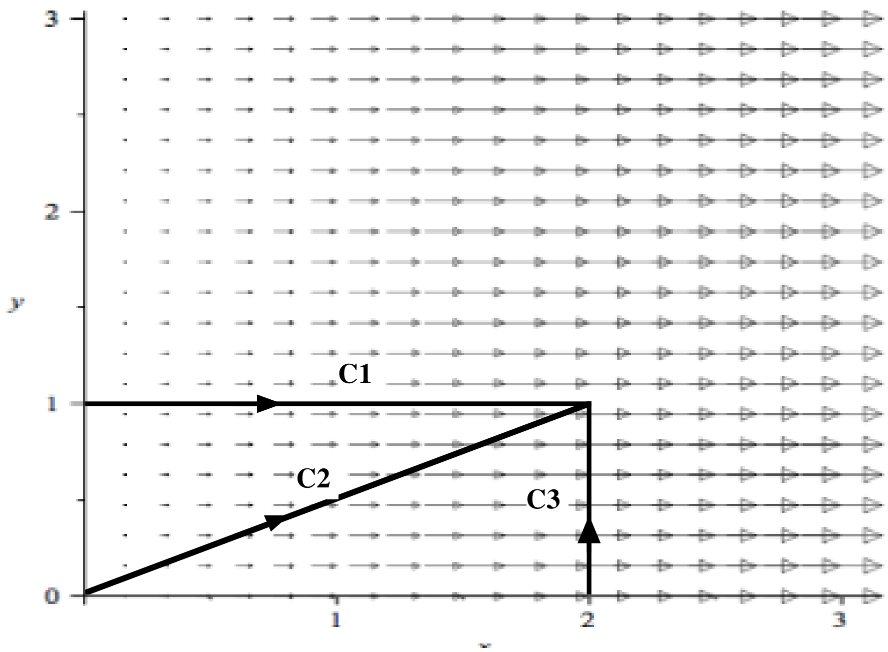


a. Consider the curve **C1**. Is $\int_{C1} \mathbf{F} \cdot d\mathbf{r}$ positive, negative, or zero? Justify your answer.

b. Consider the curve **C2**. Is $\int_{C2} \mathbf{F} \cdot d\mathbf{r}$ positive negative, or zero? Justify your answer.

c. Is **F** a conservative vector field? Justify your answer.

9. Let **F** be the vector field given in the diagram below.



a. Consider the curve **C3**. Is $\int_{C3} \mathbf{F} \cdot d\mathbf{r}$ positive, negative, or zero? Justify your answer.

b. Consider the curve **C2**. Is $\int_{C2} \mathbf{F} \cdot d\mathbf{r}$ positive negative, or zero? Justify your answer.

c. Consider the two curves **C1** and **C2**. Is $\int_{C1} \mathbf{F} \cdot d\mathbf{r}$ greater than, less than, or equal, to $\int_{C2} \mathbf{F} \cdot d\mathbf{r}$? Justify your answer.

d. Is **F** a conservative vector field? Justify your answer.