Math 213 Calculus III

May 2013
Friday, May 3
Sections 9.5-9.6
Topics:

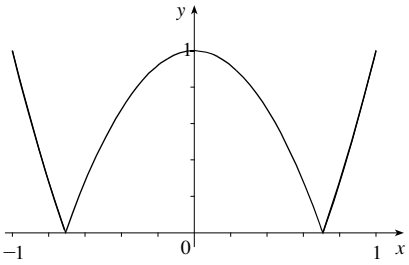
1. Three ways to describe a line:
   1. Vector (parametric) equations (starting with a point on the line and a direction vector)
   2. Symmetric equations
   3. Two point vector equation (starting with two points on the line)
2. Three ways to describe a plane:
   1. Vector equation (starting with a point on the plane and a normal vector)
   2. Scalar equation
   3. Parametric equations (starting with a point on the plane and two direction vectors)
3. A function of two variables as a rule assigning a real number to every point in its domain, and the definition and shape of the domain of such a function
4. The representation of graphs of functions of two variables as surfaces in $\mathbb{R}^3$ and the uses of horizontal traces to describe these surfaces.
5. Quadric surfaces as the graphs of second-degree polynomials in x, y, and z.

Homework for Monday
Homework Problems:

1. WebAssign Assignment 3
2. Mathematica Class 03 Surface Lab
3. All Worksheets (in class handouts)
4.

Reading the Text
Read Sections 9.7, 10.1-10.2 and answer the following questions

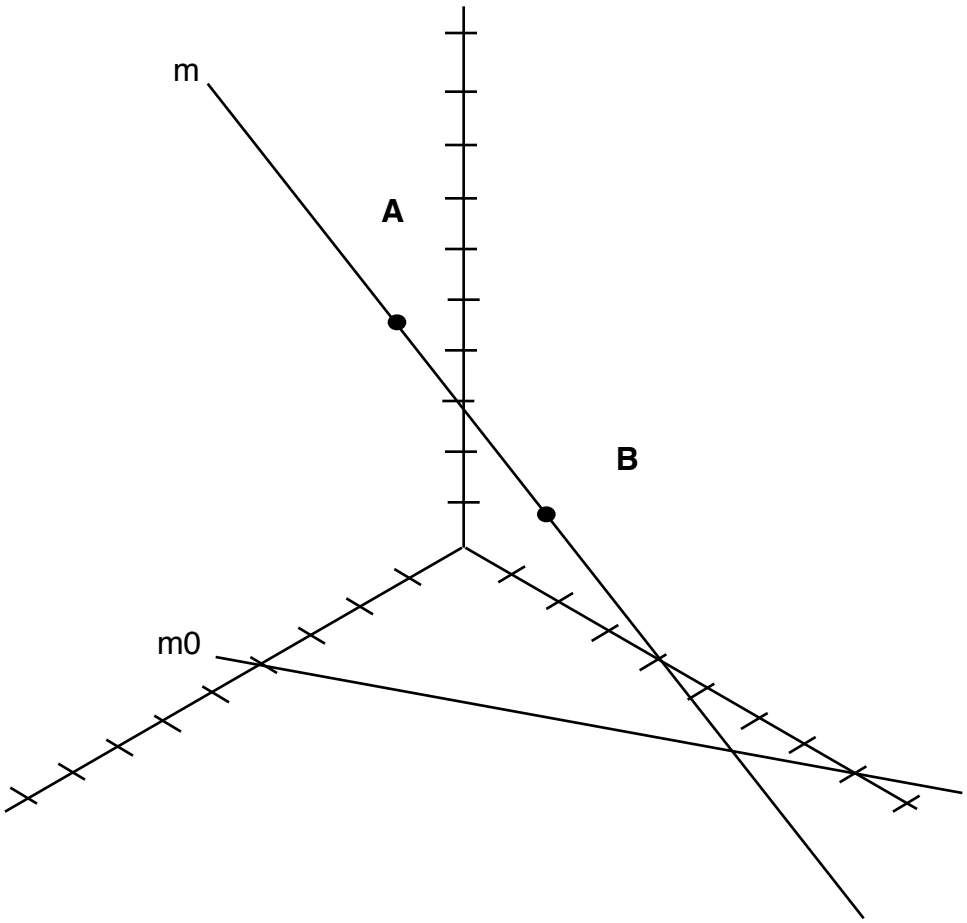
1. State a reason for introducing a different coordinate system for $\mathbb{R}^3$?
2. What surface is given by the equation $\rho = 3$?
3. The following is a graph of the parametric curve $x(t) = \sin t, y(t) = |\cos 2t|, -\pi < t < \pi$:



   Write an equation of a vector function which describes the same curve.
4. Sketch the curve described by the vector function $\langle \sin t, t \rangle$.
5. What is the difference between the *tangent vector* to a curve at a point *P* and the *unit tangent vector* to a curve at a point *P*?
6. Are there any values of *t* for which the vector function $\mathbf{r}(t) = \langle t^2 - 2t, \sin \pi t + \pi t \rangle$ has a cusp? If so, find them. If not, why not?

Math 213 Class 03: Lines in 3D Space

The line m in space is shown in the diagram below. The line m0 in the xy plane is the *projection* of the line m onto the xy plane.



1. A and B are two points on m. Find their coordinates.

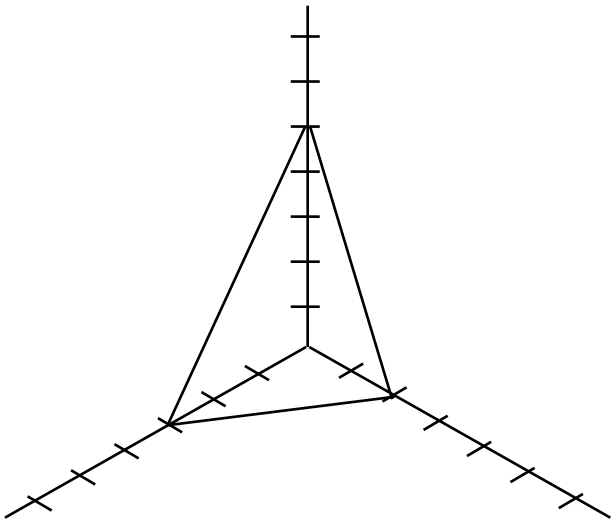
2. Draw the points E, F, and G at which the line m intersects the xy plane, the xz plane, and the yz plane, and give estimates of the coordinates of E, F, and G.

3. Using all this information, give an equation for the line m.

4. Give an equation for the line m0.

Math 213 Class 03: Planes in 3D Space

Find the equation of the plane shown below.

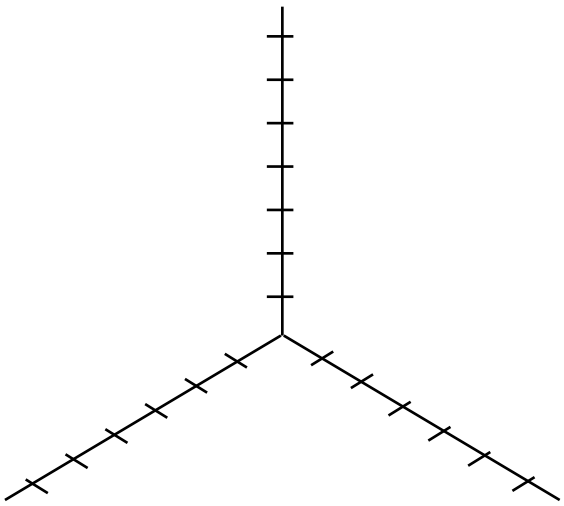


2. Draw into the xyz system above the plane whose equation is:
   $4x+2y+8z=8$

3. On the graph, show the intersection of the two planes.

4. Find the equation of the line that is the intersection of the two planes.

6. Draw the plane $x+2y=4$ into the xyz system below.



Math 213 Class 03: Planes from points

Consider the point _____ in $\mathbb{R}^3$.

Find an equation of a line that contains your point and the origin.

Find an equation of a line that contains your point and the point (1,-1,1).

Now find an equation of the plane that contains the two lines you've just found.

Find an equation of the plane that contains your point and is perpendicular to the *x* axis.

Find an equation of the plane that contains your point and is perpendicular to the line $y = x$ in the *xy* plane.

Finally, find an equation of a plane that does *not* contain your point.

Math 213 Class 03: Staying Cool

Let $T(x, y)$ be the temperature in a 10 ft by 10 ft room on a winter night, where one corner of the room is at (0,0) and the opposite corner is at (10,10). For each of the following functions T,

(a) Draw or describe in words a graph of the temperature function.
(b) Describe the likely floor locations of the heating vents.
(c) Suppose you like to sleep with a temperature of 70° or less. Where would you put the bed?

1. $T(x,y) = 78 - \frac{1}{10}\left[x^2 + (y-5)^2\right]$

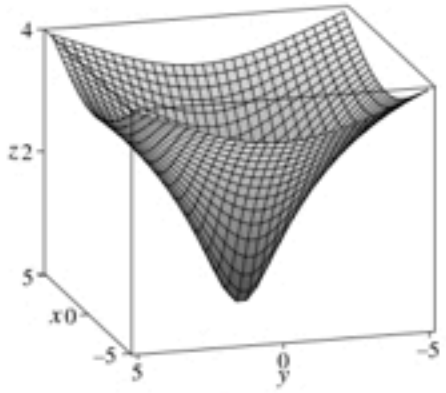
2. $T(x,y) = \frac{1}{2}x - y + 75$

Math 213 Class 03: Matching

Group Work 2, Section 9.6
The Matching Game (General Functions)

Match each function with its graph. Give reasons for your choices.

1. $f(x,y) = \dfrac{1}{x+1} + \sin y$
2. $f(x,y) = \sqrt{4 - x^2 - y^2}$
3. $f(x,y) = \cos\left(x + y^2\right)$
4. $f(x,y) = \ln\left(x^2 + y^2 + 1\right)$
5. $f(x,y) = x^2\sqrt{y}$
6. $f(x,y) = x^3 y$



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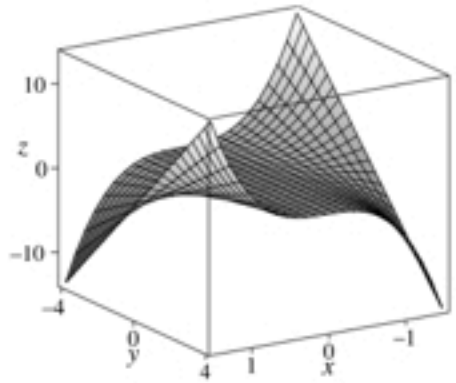

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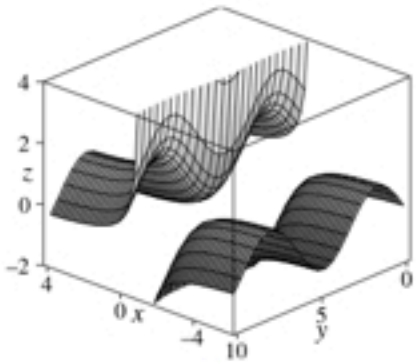

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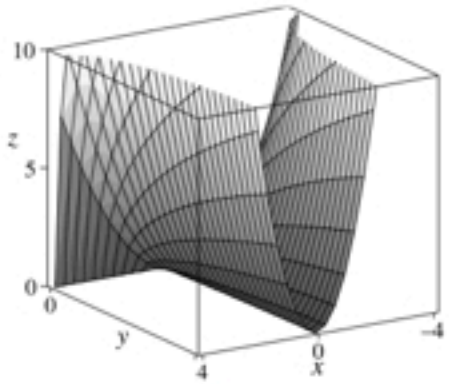

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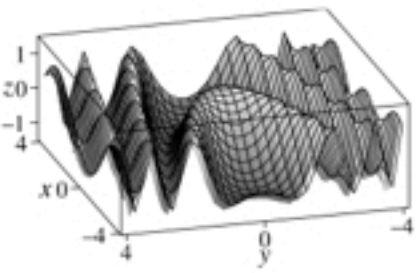

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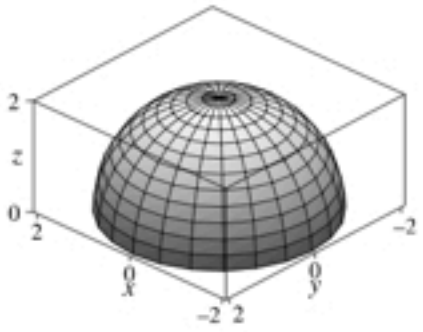

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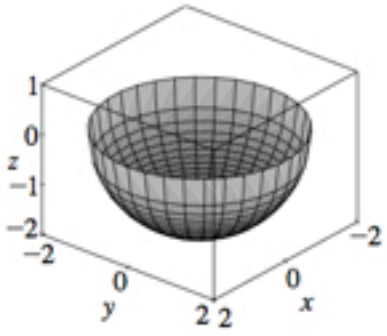
Math 213 Class 03: Matching

Group Work 3, Section 9.6
The Matching Game (Quadric Surfaces)

Match each function with its graph. Give reasons for your choices.

1. $x^2 + y^2 + \frac{1}{4}z^2 = 1$
2. $z = -\sqrt{4 - x^2 - y^2}$
3. $y^2 + \frac{1}{4}z^2 = 1$
4. $\frac{1}{9}z^2 - \frac{1}{4}y^2 = 1$
5. $\frac{1}{4}x^2 - y^2 - z^2 = 1$
6. $|z| = \sqrt{x^2 + y^2}$



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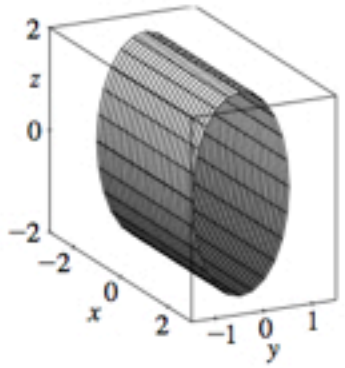

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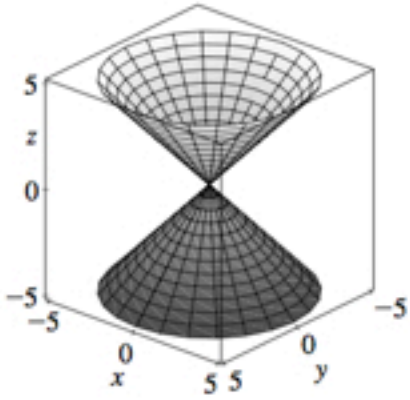

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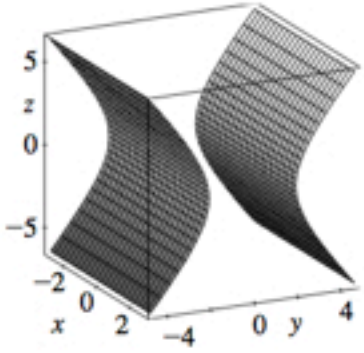

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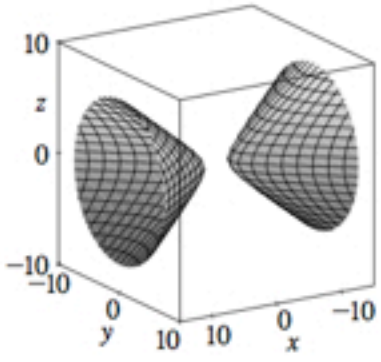

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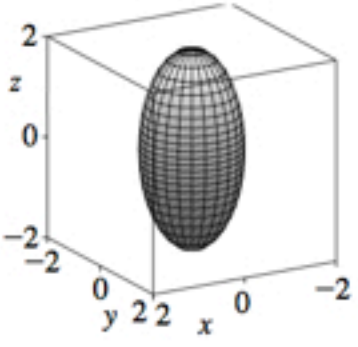

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