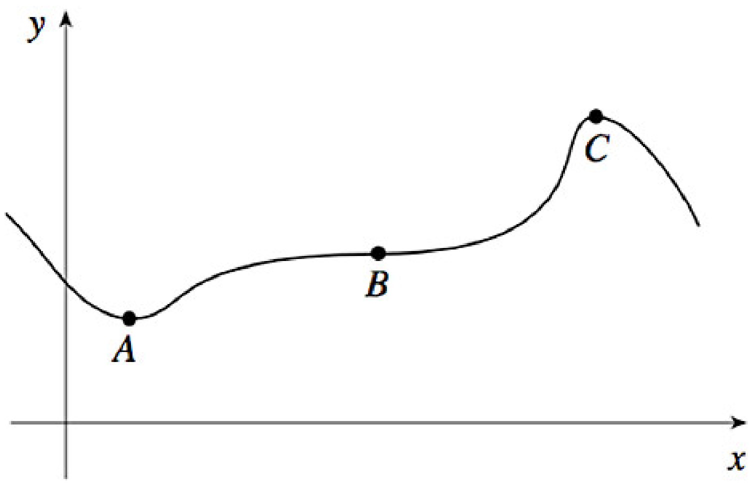
Math 213 - May 2024 - Test 1 NAME __MAX____

On this exam, I'll write vector quantities in a bold-face, non-italic font like: $\mathbf{v}$, $\mathbf{a_T}$. Scalar quantities will be in a non-bold, italic font, like this: s, m_x. I'll write unit vectors as vectors with a caret overtop, like this: $\hat{\mathbf{k}}$, $\hat{\mathbf{T}}$.

1. Consider the vector function $\mathbf{r}(t)$ describing the 2-d curve shown below, and the curvatures of the curve at positions at A, B, and C. Put the positions in order from smallest curvature to largest.



<-- smallest

<--

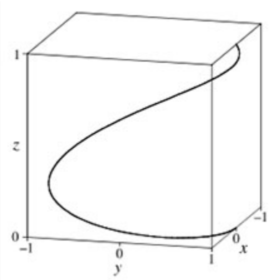
<--largest

2. Consider two vectors, $\mathbf{a} = \langle -5, 1, 1 \rangle$ and $\mathbf{b} = \langle 1, 1, 2 \rangle$.
   a. What's the angle between $\mathbf{a}$ and $\mathbf{b}$? (Give your answer in both radians and degrees rounded to 3 digits.)

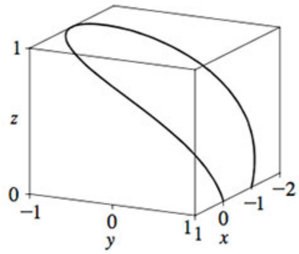
   b. Find a vector, $\mathbf{v}$, which is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

3. Let $\mathbf{a} = (u+v)\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, and $\mathbf{b} = 2\hat{\mathbf{i}} + (2u+3v+1)\hat{\mathbf{j}} + \hat{\mathbf{k}}$. Find a value for u and a value for v such that $\mathbf{a} \perp \mathbf{b}$.

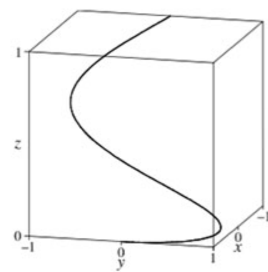
4. Which curve below is the path traced out by $\mathbf{r}(t) = \langle \sin \pi t, \cos \pi t, \frac{1}{4}t^2 \rangle$, $0 \leq t \leq 2$? Justify your answer.



Graph 1



Graph 2



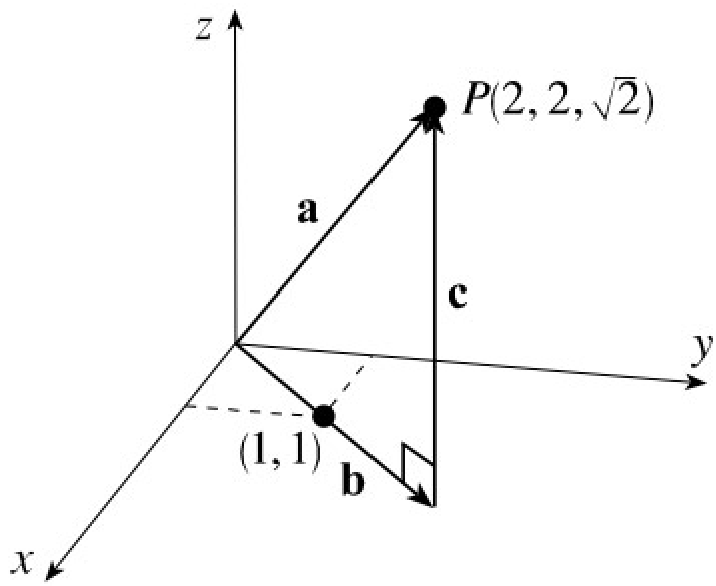
Graph 3

5. Using the point $P = (0, -2, 3)$ and a vector $\mathbf{b} = \langle 3, -2, 1 \rangle$:

   a. Find an equation for the plane which is normal to $\mathbf{b}$ and includes the point P. The equation should be in the form $c_1 x + c_2 y + c_3 z = c_4$.

   b. *Now consider a line, L, which is parallel to $\mathbf{b}$, and passes through P. What are the coordinates of the point at which this line pierces the xy plane?*

6. *Let* $\mathbf{a} = \overrightarrow{\mathbf{OP}}$ *where* P *is the point* $(2, 2, \sqrt{2})$.



a. *Find the components of vectors* $\mathbf{b}$ *and* $\mathbf{c}$. *(The tail of* $\mathbf{b}$ *is at the origin.)*

b. *Find the distance of the point* P *from the origin.*

c. *Find the distance of the point* P *from the* z*-axis.*

7. *A crazed ostrich ("Oscar") runs along a mountain path according to* $\mathbf{r}(t) = \langle 3e^{-t}, 4(\cos t + t), t^2/18 \rangle$, *with positions in meters and time in seconds.*

a. *What is the change in Oscar's altitude (*z*-component) from t = 0 to t = 6?*

b. *What is Oscar's speed at* t *= 0 seconds?*

c. *Find a formula (but don't try to solve the integral!) for the total distance that Oscar travels from* $t = 0$ *to* $t = 6$.

8. *Beside each of the surface plots below, make a sketch of the trace* $z(y) = z(x = 0, y)$ *Then, match the equations with their graphs. Give a reason for your choices.*

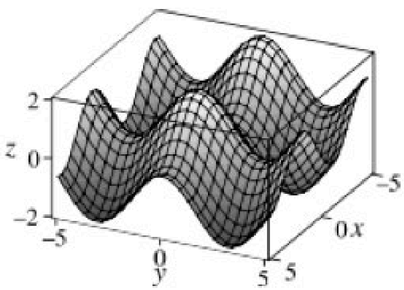
(a) $8x + 2y + 3z = 0$

(b) $z = \sin x + \cos y$

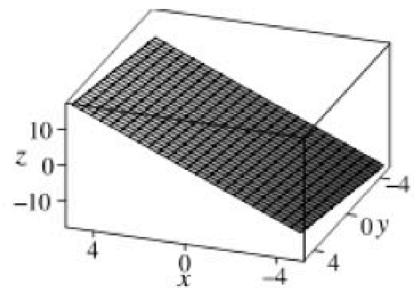
(c) $z = \sin\left(\dfrac{\pi}{2 + x^2 + y^2}\right)$

(d) $z = e^y$

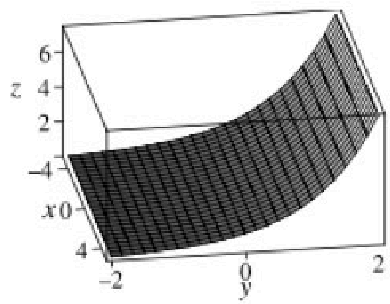
I



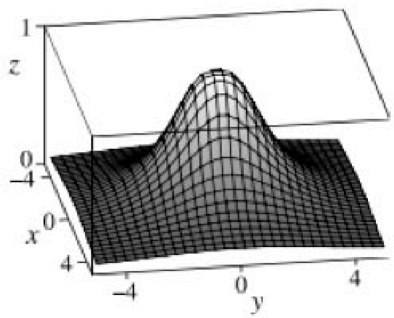
II



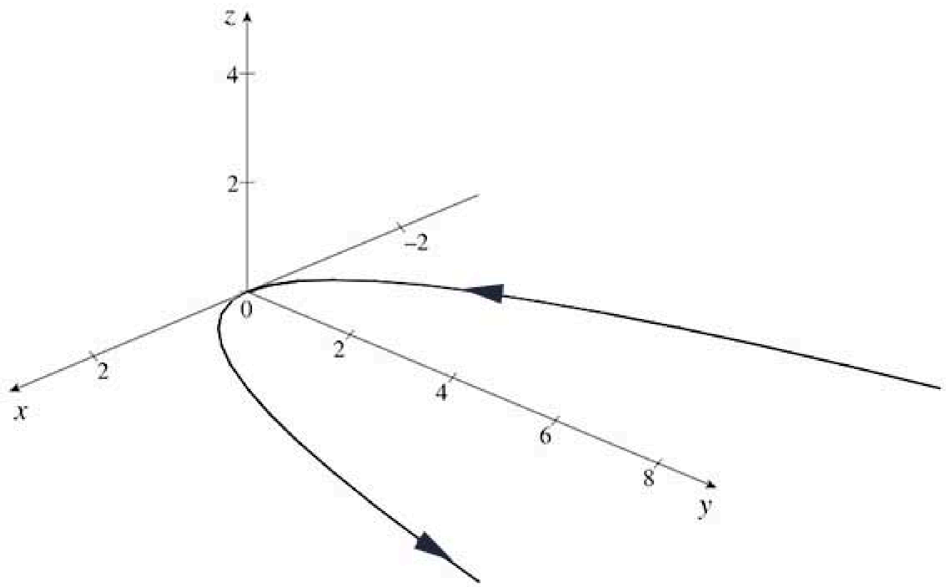
III



IV



9. *Let* $y = x^2$ *be the parabolic trajectory of a particle in the* xy *plane parameterized by* $\mathbf{r}(t) = \langle t, t^2, 0 \rangle$*. This is shown below, as well as an arrow indicating the direction in which the particle traverses the trajectory.*



a. *Compute* $\mathbf{r}'(t)$ *and* $\mathbf{r}''(t)$

b. *What is the unit tangent vector,* $\hat{\mathbf{T}}$*, at the origin (0,0,0)? (No complicated computations are needed.) Give the components of* $\hat{\mathbf{T}}$*, as well as sketching it on the diagram.*

c. *Give the vector form of a line which is tangent to the curve when* $t = 2$*, and sketch the line on the diagram.*

This page is left blank for you to use as scratch paper. If you would like any work here to count for partial credit, label it with the question number.