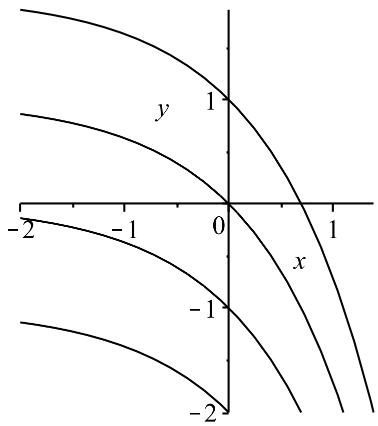
Name:

Math 213 May 2014 Test 2 **Wednesday, May 14, 2014**

1. Several level curves for the function $f(x,y) = e^x + y$ are shown below.



(a) What is the value of $f(x,y)$ on the each of the (not necessarily equally spaced) level curves shown? Label each level curve with the value of $f(x,y)$ ***on*** that level curve.

(b) Find a formula for the level curve that passes through the point $(0,1)$.

(c) At the point $(0,1)$ draw and label the vector $\nabla f(0,1)$. Be accurate as to the direction and length of this vector.

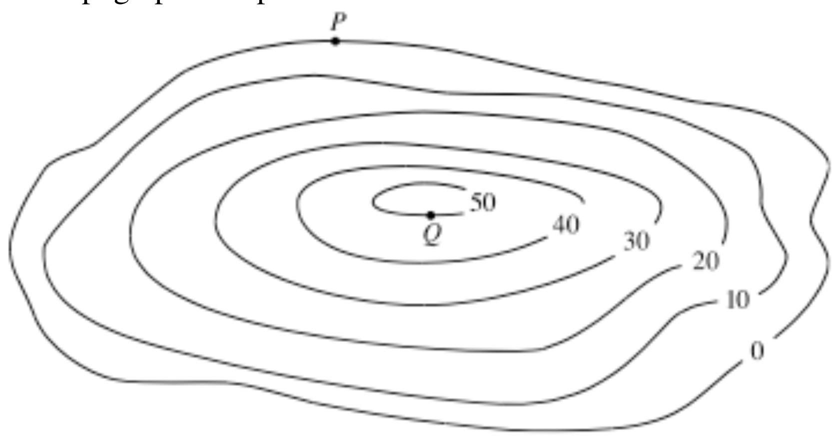
(d) On the graph above draw and label a vector that starts at $(0,1)$ and points in the direction in which $f(x,y)$ remains constant. What are the components of the vector you found?

(e) Find the directional derivative of $f(x,y)$ at the point (0,1) in the direction of

$\vec{\mathbf{v}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

(f) What is the maximum possible value of the directional derivative at (0,1)?

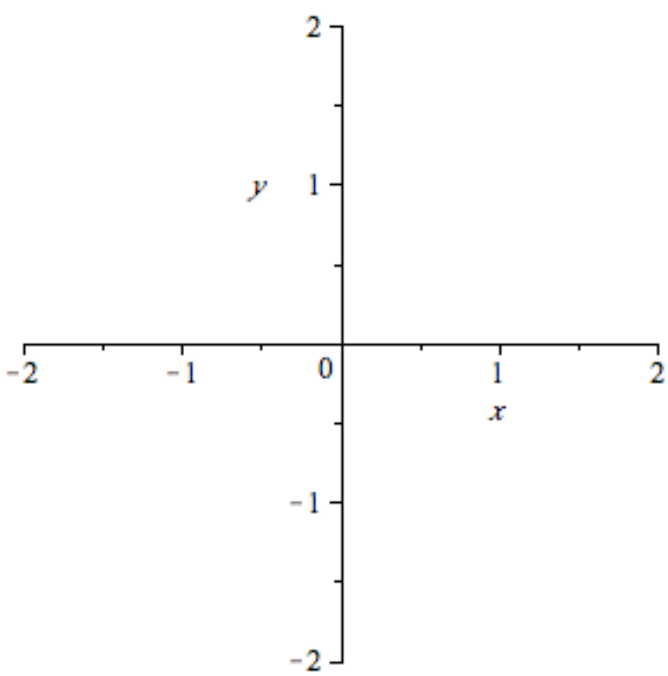
2. Below is a topographic map of a hill.



(a) Starting at point P, sketch the path of steepest ascent to the peak elevation of 50 yards.

(b) Suppose it rains and water runs down the hill starting at point Q. Mark the point at which you would expect the water to reach the bottom (height=0). Justify your answer.

3. Draw a contour map for the function $f(x,y) = x + 2y - 1$. with at least 4 labeled contours.



4. A metal plate is situated in the *xy* plane and occupies the rectangle $0 \le x \le 10, 0 \le y \le 8$ where *x* and *y* are measured in meters. The temperature at the point (x,y) in the plate is $T(x,y)$ where *T* is measured in degrees Celsius. Temperatures at equally spaced points were measured and recorded. A portion of those measurements is shown in the table.

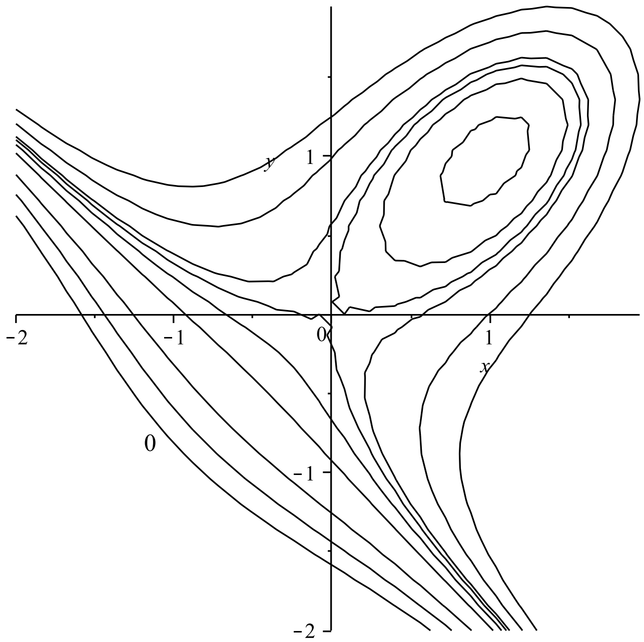
	y=2	y=4	y=6
x=4	60	62	61
x=6	72	68	66
x=8	80	75	71

(a) Estimate the values of the partial derivatives $T_x(6,4)$ and $T_y(6,4)$. Give ***units*** with your answers.

(b) Estimate the value of $T(7,5)$. Show your work.

5. Consider the surface $f(x,y) = 4 + x^3 + y^3 - 3xy$. A contour plot for this function is shown.

(a) Find f_x and f_y.



(b) Find the critical points for this function.

(c) Determine whether each critical point is a local and/or global maximum, local and/or global minimum, or saddle point.

(d) Label some point on the graph where $f_{xy} < 0$ if possible.
Label some point on the graph where $f_{xy} > 0$ if possible.

6. Find the extreme values of $f(x,y) = 9 - x^2 - y^2$ subject to the constraint $x + y = 3$ by performing the following steps.

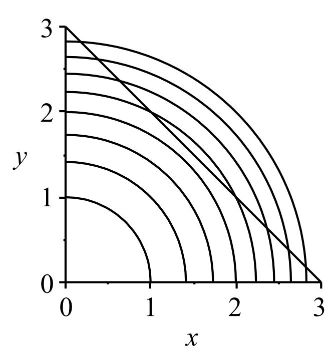
(a) Give the Lagrange equations that need to be satisfied.

(b) Solve the Lagrange equations. (There should be a unique critical point, P.)

(c) Find the value of $f(x,y)$ at the point P you found in part (b)

(d) Determine if P corresponds to a minimum or maximum value of f.

Hint: Do the values of $f(x,y)$ increase or decrease as (x,y) moves away from P along the line $x + y = 3$? The figure shows some level curves of $f(x,y)$ and the constraint $x + y = 3$.



7. Consider the integral $\int_0^2 \int_{x^2}^{2x} \left(x^2 + 4y\right) dy dx$.

(a) Sketch the region of integration, labeling all important points.

(b) Evaluate the integral by hand, showing all steps in the computation, including antiderivatives.

(c) Write the integral that reverses the order of integration. (You do not need to evaluate it.)