Name____

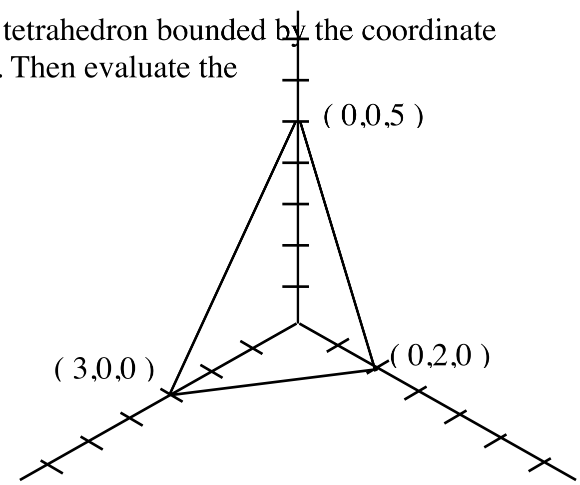
Math 213 May 2013 Test 3 **Wednesday, May 22, 2013**

Instructions: Show **ALL** *your work. Correct answers without justification will be given* **NO** *credit.*

1. Consider the triple integral $\int_0^1 \int_{y^3}^{\sqrt{y}} \int_0^{xy} dz\,dx\,dy$ representing the volume of a solid S. Let R be the projection of S onto the plane $z = 0$.
   (a) Carefully draw the region R on the xy plane (labeling ***all*** important features).

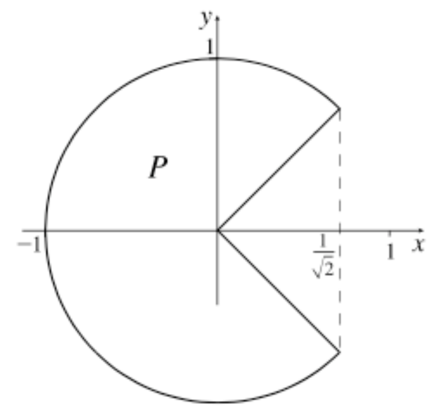
   (b) Rewrite this integral as $\iiint_S dz\,dy\,dx$ (note the different integration order.)

2. Set up a *triple* integral to evaluate the volume of the tetrahedron bounded by the coordinate planes and the plane $10x + 15y + 6z = 30$, as shown. Then evaluate the integral.



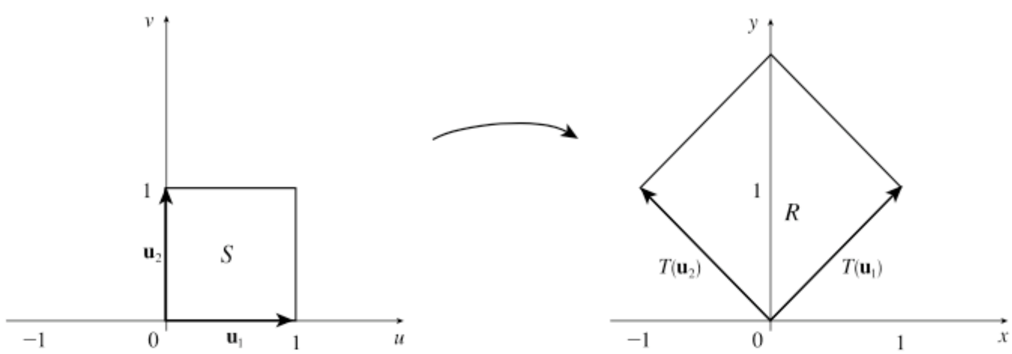
3. Consider the following region in the shape of Pac-Man.

Using polar coordinates, set up the integral to evaluate $\iint_{Pac-Man} x\,dA$. Then evaluate the integral.



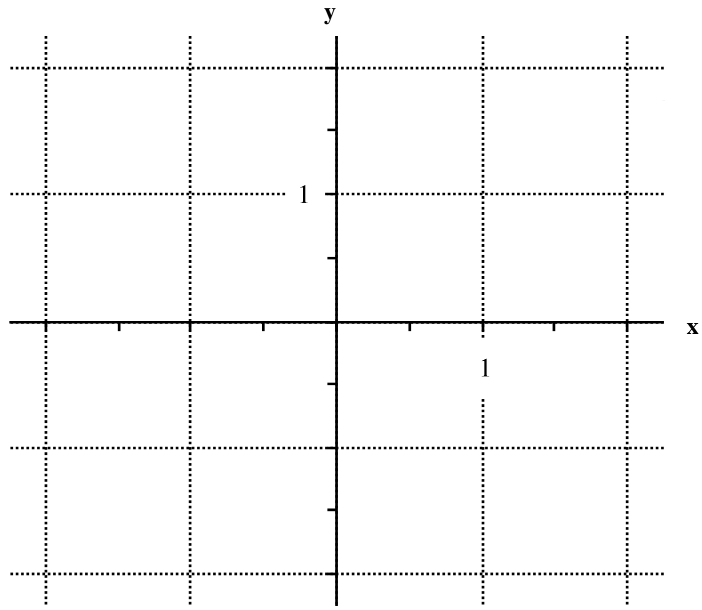
4. Is it true that $\int_0^1 \int_x^1 f(x,y)\,dydx = \int_0^1 \int_y^1 f(x,y)\,dxdy$? Explain.

5. The linear transformation $\begin{aligned} x &= u - v \\ y &= u + v \end{aligned}$ takes the unit square S in the uv plane to the square shown in the xy plane.



Calculate the Jacobian of the transformation and explain its significance.

6. Let $\mathbf{F}(x,y) = 2y\,\mathbf{i} + \mathbf{j}$. Carefully sketch several representative vectors at the following points: (0,0), (1,0), (–1,0), (0,1), (0.–1), (1,–1), (–1,1).



7. Find a vector function $\mathbf{F}$ that has the potential function $f(x,y,z) = x^2e^{yz}$.

8. Let $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + (y^2 - 2z)\,\mathbf{j} + \sin(yz)\,\mathbf{k}$

(a) Compute the divergence of $\mathbf{F}$.

(b) Compute the curl of $\mathbf{F}$.

9. Let C be the line segment from the point $(1,0,2)$ to the point $(3,4,1)$. Show ***all*** your work.

(a) Given $\mathbf{F} = 2x\,\mathbf{i} + \left(y^2 + z\right)\mathbf{j} + (x + y)\,\mathbf{k}$, find the value of $\int \mathbf{F} \cdot d\,\mathbf{r}$ along the line segment.

(b) Find $\int_C xyz^2 ds$ along the line segment.

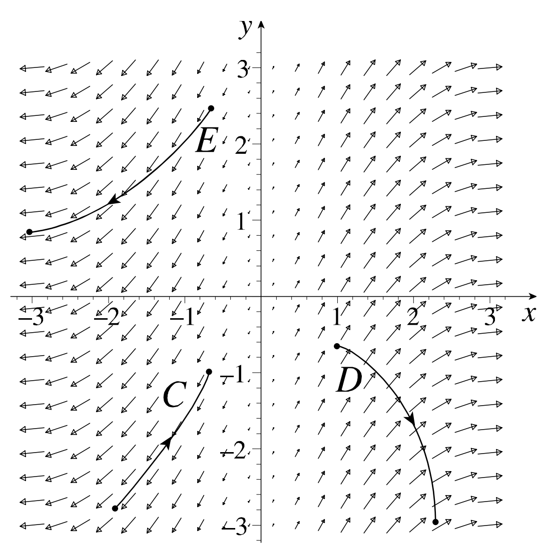
10. Let $\mathbf{F}(x,y) = (3 + 2xy)\,\mathbf{i} + \left(x^2 - 3y^2\right)\mathbf{j}$.

(a) Use an appropriate test to show that $\mathbf{F}$ is a conservative vector field.

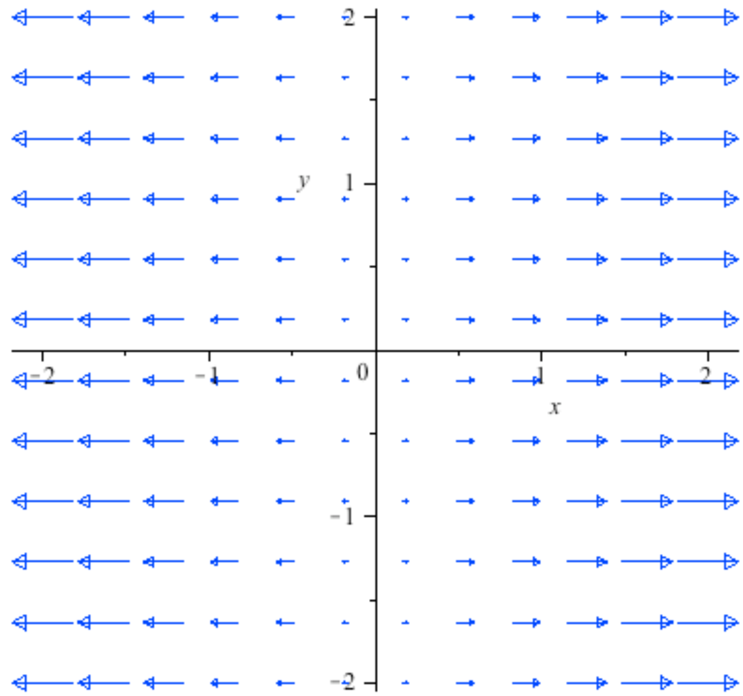
(b) Find a potential function for $\mathbf{F}$.

(c) Let C be the curve by $\mathbf{r}(t) = (-\cos t - 1)\,\mathbf{i} + (-\sin t)\,\mathbf{j}$ with $0 \le t \le \pi$. Using your answers from (a) and (b), indirectly compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

11. Place the three line integrals $\int_C \mathbf{F} \bullet d\mathbf{r}$, $\int_D \mathbf{F} \bullet d\mathbf{r}$, $\int_E \mathbf{F} \bullet d\mathbf{r}$ in order from largest (most positive) to smallest (most negative).



12. Let **F** be the vector field given in the diagram to the right.



(a) What can you say about the curl of **F**?

(b) What can you say about the divergence of **F**?

(c) Is **F** a conservative field? Why or why not?

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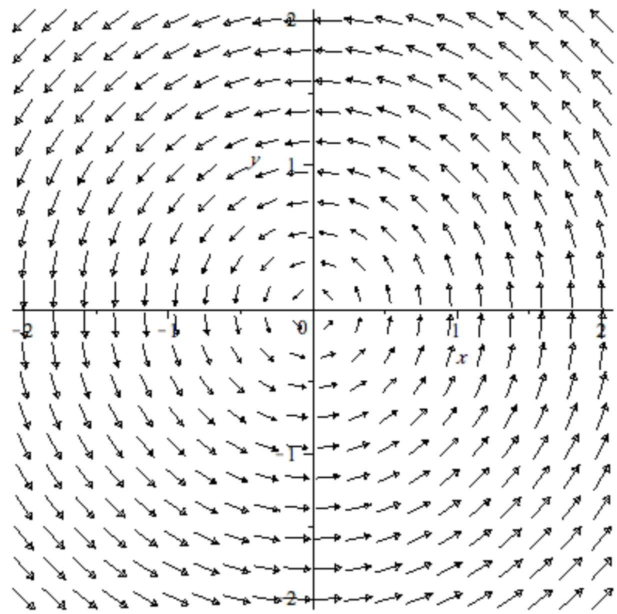
13. Consider the vector field shown to the right. Draw ***and label*** curves that go from $(-2,0)$ to $(2,0)$ such that



(a) $\int_C \mathbf{F}\cdot d\mathbf{r} > 0$

(b) $\int_C \mathbf{F}\cdot d\mathbf{r} < 0$

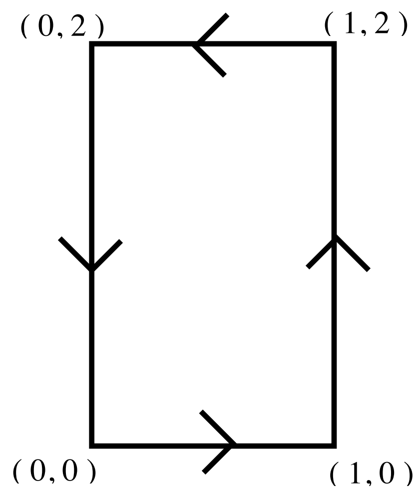
(c) $\int_C \mathbf{F}\cdot d\mathbf{r} = 0$

(***Note***: indicate direction along the curves)

14. Use Green's Theorem to compute the $\int_C \mathbf{F}\cdot d\mathbf{r}$ where

$\vec{\mathbf{F}}(x,y) = \left(\sin x + xy^2\right)\hat{\mathbf{i}} + \left(e^y + \frac{1}{2}x^2\right)\hat{\mathbf{j}}$ and where C is the path shown below.

(Write the expression for $\int_C \mathbf{F}\cdot d\mathbf{r}$ ***and*** the expression you get by applying Green's Theorem.)



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