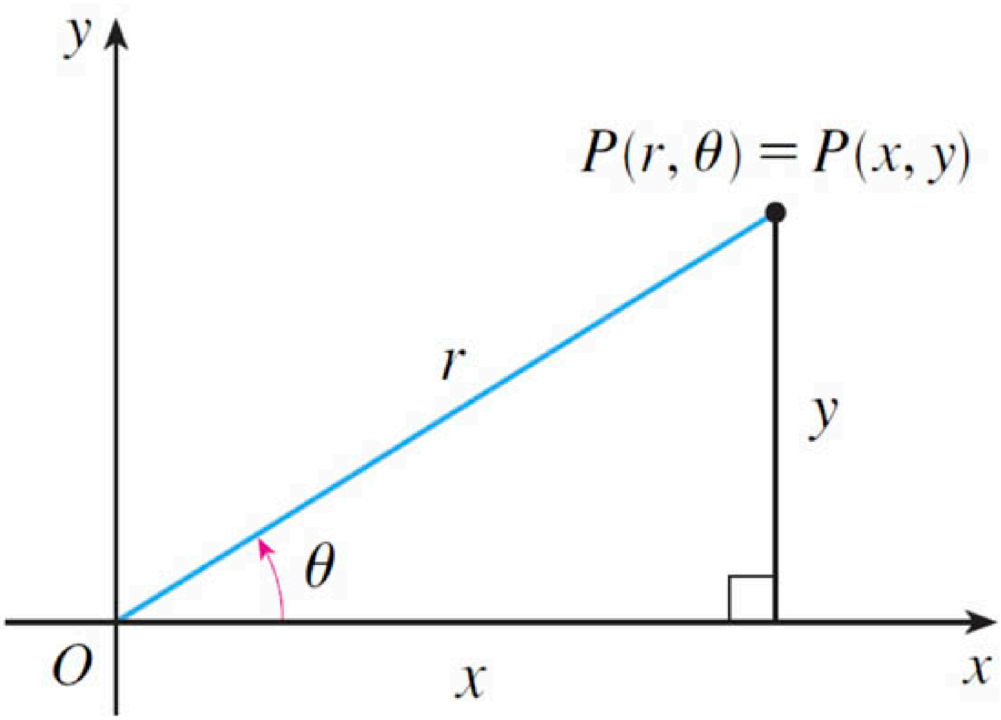
Cylindrical and Spherical Coordinates [9.7]

Polar coordinates (2-d)



$(x, y) \to (r, \theta)$:

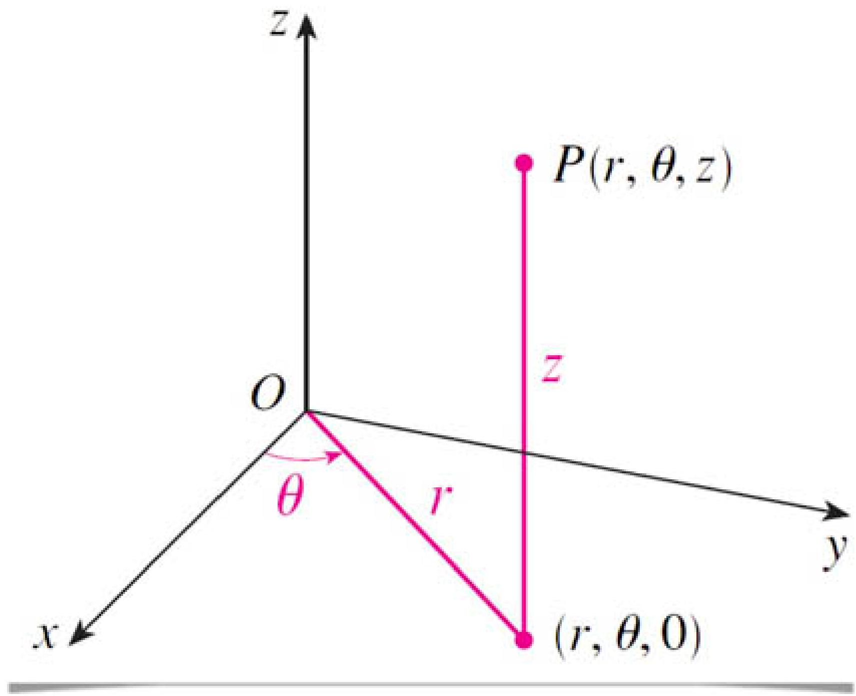
$$r^2 = x^2 + y^2; \quad \tan\theta = \frac{y}{x}. \tag{1}$$

$(r, \theta) \to (x, y)$:

$$x = r\cos\theta; \quad y = r\sin\theta. \tag{2}$$

Cylindrical coordinates (3-d)

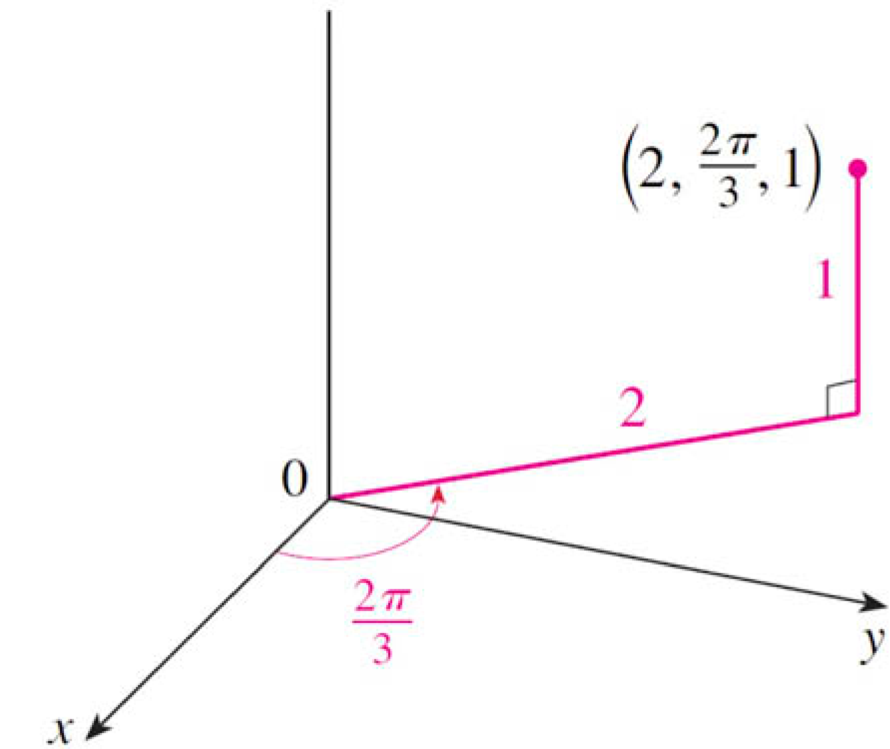
Polar coordinates in the $x - y-$plane, plus z which is the same in cylindrical and Cartesian coordinates.



For example

$(r, \theta, z) = (2, \frac{2\pi}{3}, 1)$:

- $x = r\cos\theta = 2(-\frac{1}{2}) = -1$
- $y = r\sin\theta = 2(\frac{\sqrt{3}}{2}) = \sqrt{3}$
- $z = z = 1$



N.B. **If an angle measure doesn't explicitly have the degree mark, ^{o}, it's in radians**!

All the computer / math packages I've used, work primarily in radians.

Converting

$$\pi \text{ radians} = \frac{1}{2} \text{ rotation} = 180^o. \tag{3}$$

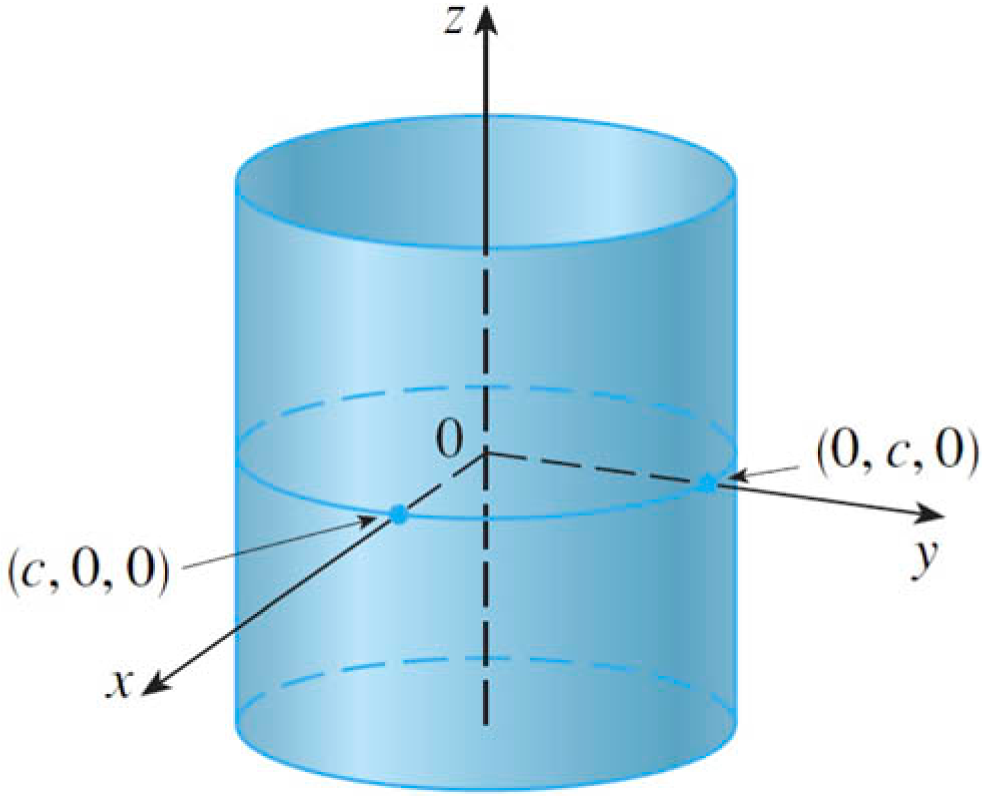
So, for example, $\pi/6$ radians...

$$\frac{\pi}{6} \text{ radians} \cdot \frac{180^o}{\pi \text{ radians}} = \frac{180^o}{6} = 30^o. \tag{4}$$

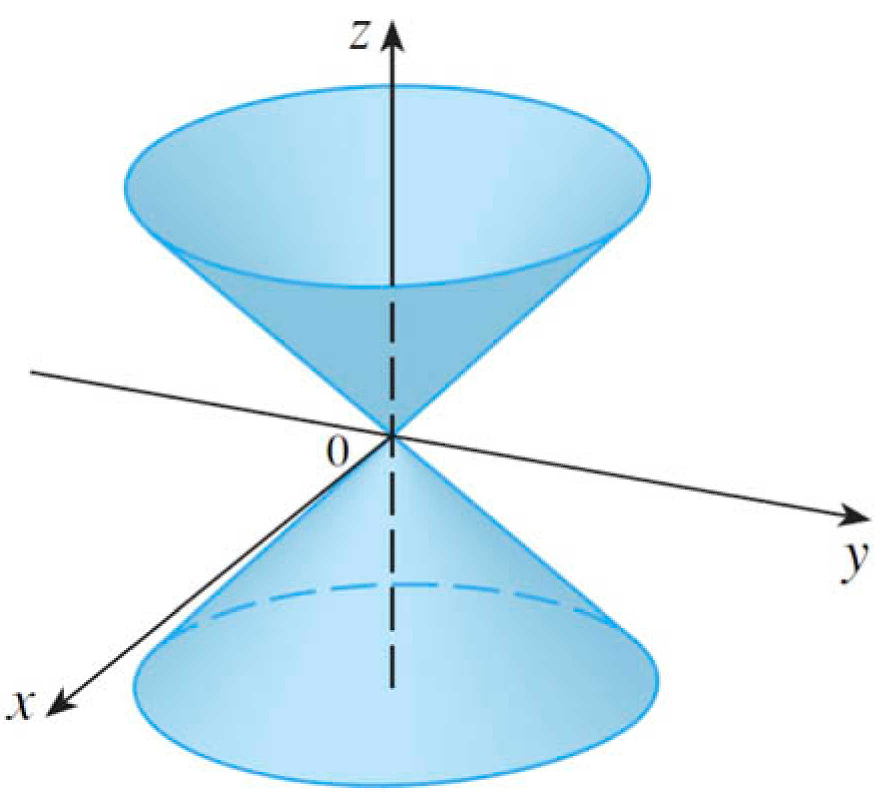
Some surfaces

...which are easily specified in cylindrical coordinates

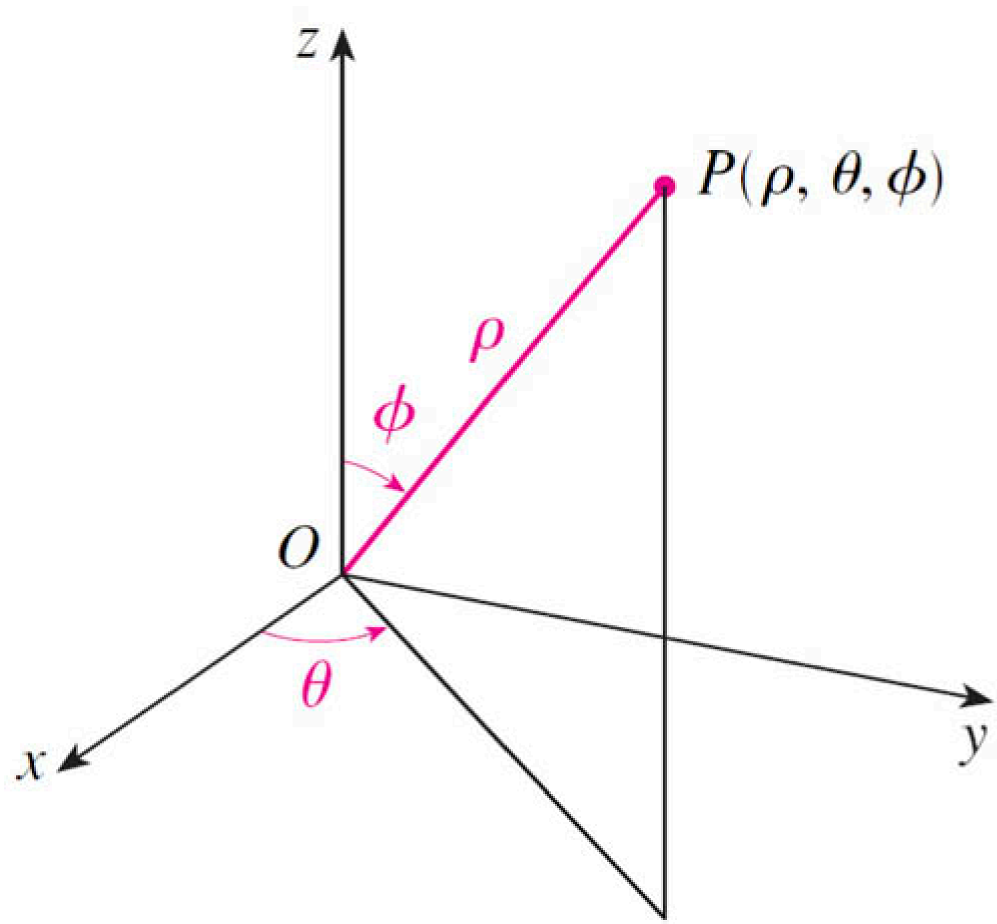
$$r = c$$



$r = z$



Spherical coordinates (3-d)



- ρ - "rho" - radial distance
- θ - "theta" - **azimuthal** angle
- ϕ - "phi" - **polar** angle

This is the convention used by publishers of calculus textbooks, and Wolfram's own *MathWorld* website.

However, physicists, engineers, *Mathematica* (in some contexts), *MatLab*, and even applied mathematicians use a [different convention](), where θ is the polar angle and ϕ is the azimuthal angle.

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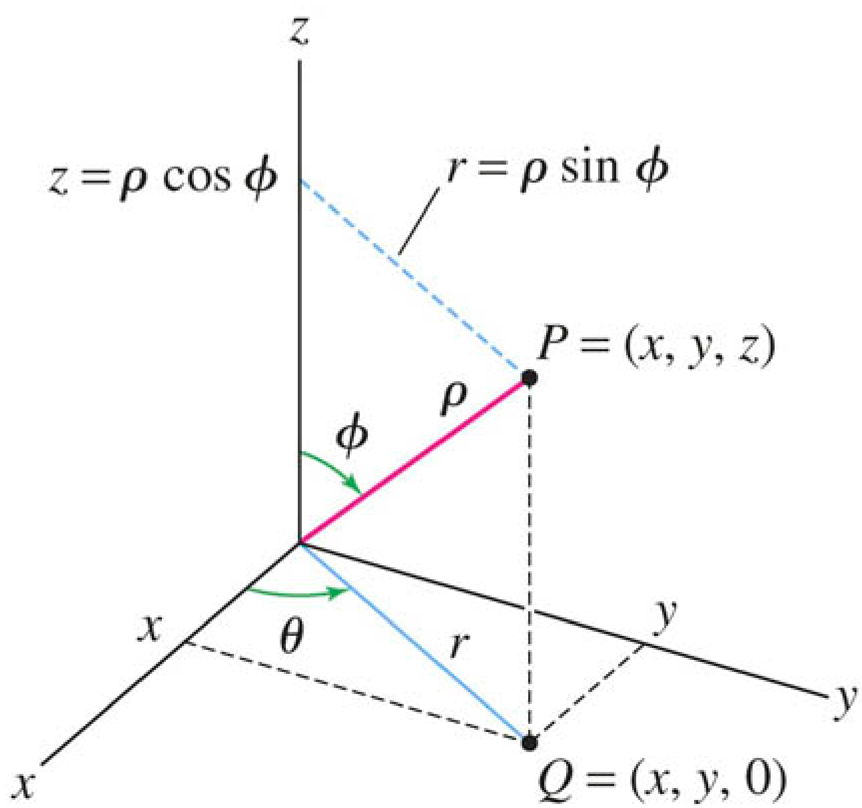
In this course, we'll use the calc-textbook convention:



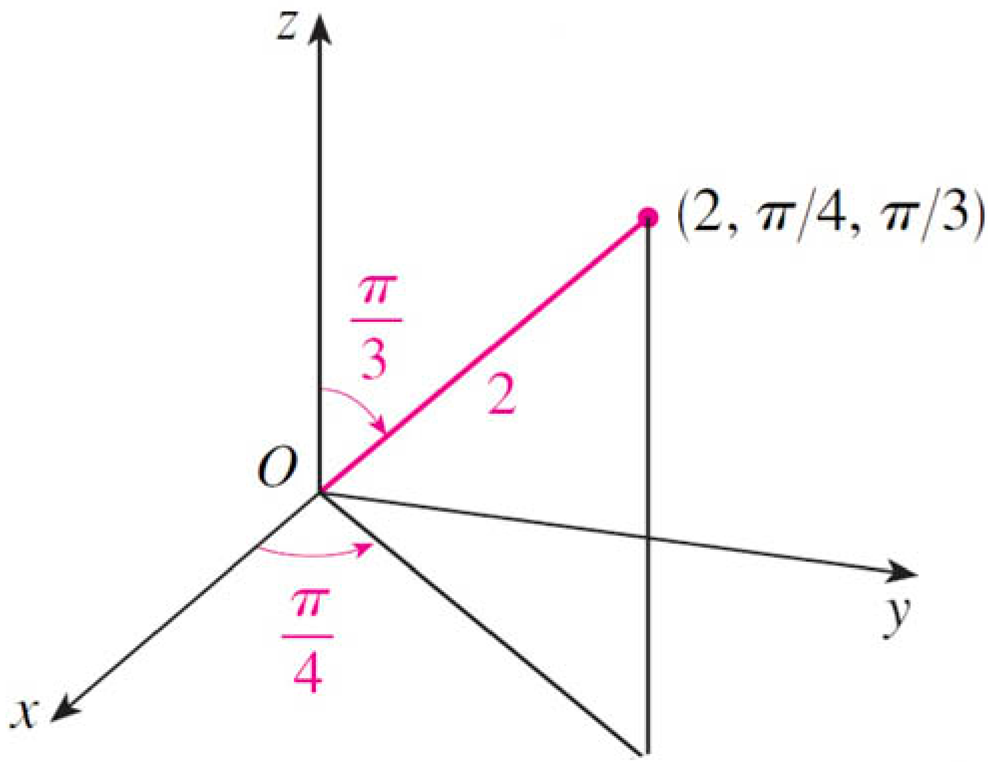
Use $r \equiv$ projection of ρ into $x-y-$plane$=\rho \sin \phi$:

- $x = r\cos\theta = \rho \sin\phi \cos\theta.$
- $y = r\sin\theta = \rho \sin\phi \sin\theta.$
- $z = \rho\cos\phi.$

$(x, y, z) \to (\rho, \theta, \phi)$

- $\rho^2 = x^2 + y^2 + z^2.$
- $\tan\theta = \frac{y}{x}.$
- $\cos\phi = \frac{z}{\rho}.$

Example

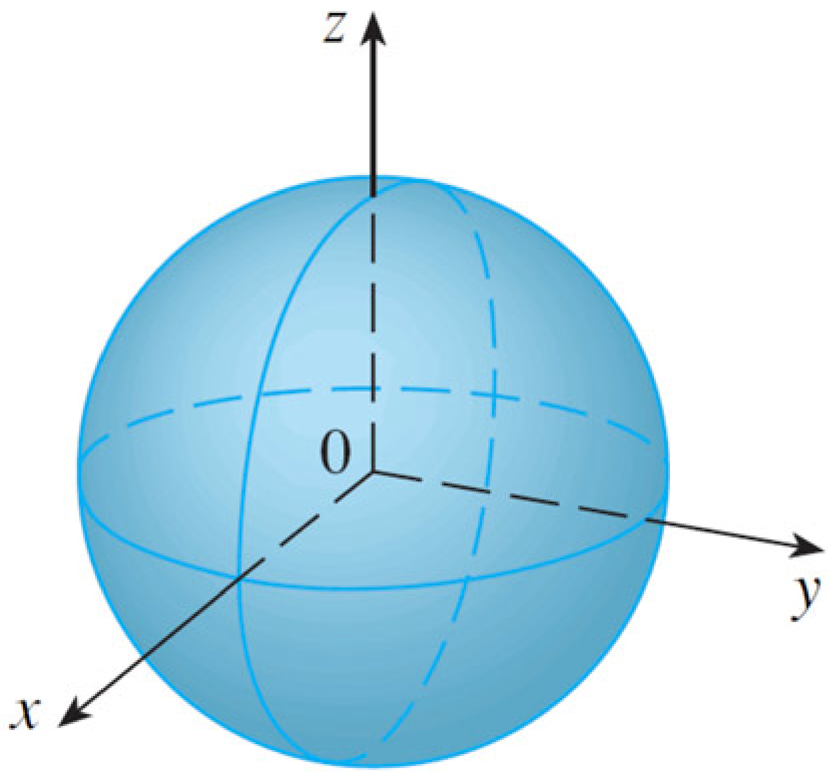


$(\rho, \theta, \phi) = (2, \pi/4, \pi/3) \to (x, y, z)$?

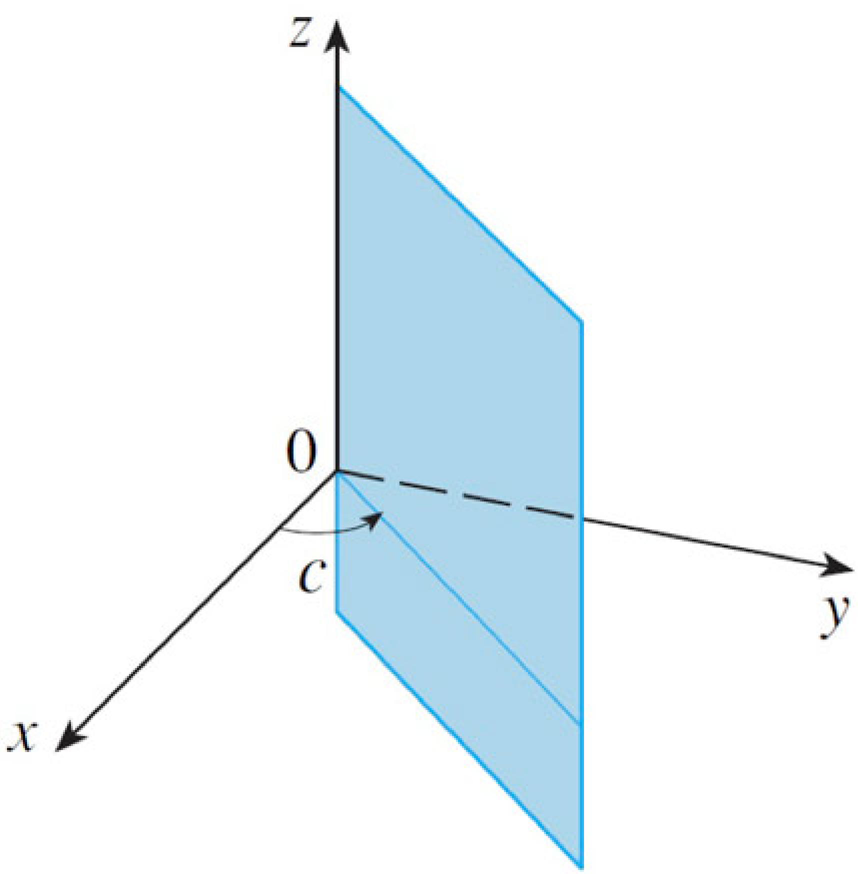
- $z = \rho \cos \frac{\pi}{3} = 1$
- $r = \rho \sin \frac{\pi}{3} = \sqrt{3}$
- $x = r \cos \frac{\pi}{4} = \sqrt{3/2} \approx 1.22$
- $y = r \sin \frac{\pi}{4} \approx 1.22$

Surfaces in spherical coordinates

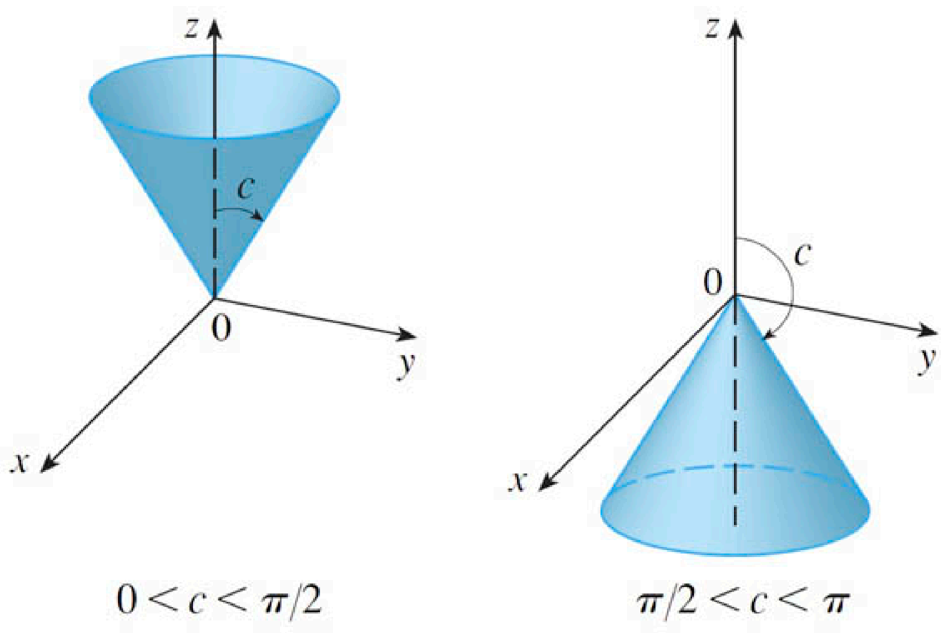
$$\rho = c \tag{5}$$



$$\theta = c \tag{6}$$



$$\phi = c \tag{7}$$

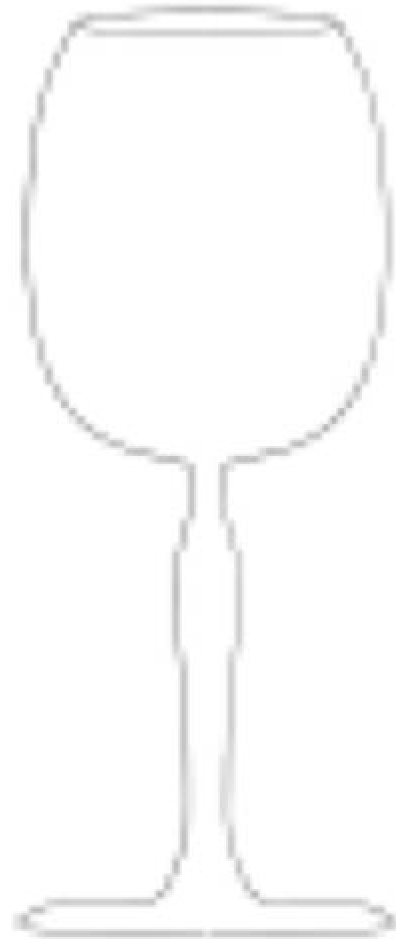


- Animation at mathinsight.org
- In GeoGebra, you can plot a `Surface( [x function], [y function], [z function], .....)` parametrically in terms of two parameters. For example, a sphere.

To do

- ***Describe me***:

  Come up with an expression (in cylindrical coordinates) to approximately describe the surface of a goblet:



Find an equation (well, actually several. One for each vertical piece of the goblet) for $r(z)$. And then, if you don't specify θ, it can take on any possible value. This is produces a "surface of rotation".

To actually code this in Geogebra, you'll use the `Surface(...)` function, which allows you to specify a set of points, using two parameters.

For example, a crude base to the goblet, might be to specify a straight cross section, (I'm thinking in millimeters):

$$r(z) = 10 - 10z \text{ for } 0 < z < 1 \tag{8}$$

and then "rotate" this line, by letting θ run from 0 to 2π.

In the (x, y, z) coordinates that Geogebra wants, you'd have:

$$x(\theta, z) = r(z)cos(\theta); \tag{9}$$

$$y(\theta, z) = r(z)sin(\theta); \tag{10}$$

$$z(theta, z) = z \tag{11}$$

Here's the example.

You'll plot this in today's lab

(Further detail about how to "hand in" this assignment in the "Space Curves and Surfaces" notebook file for today.)

Image credits

Bethan Phillips