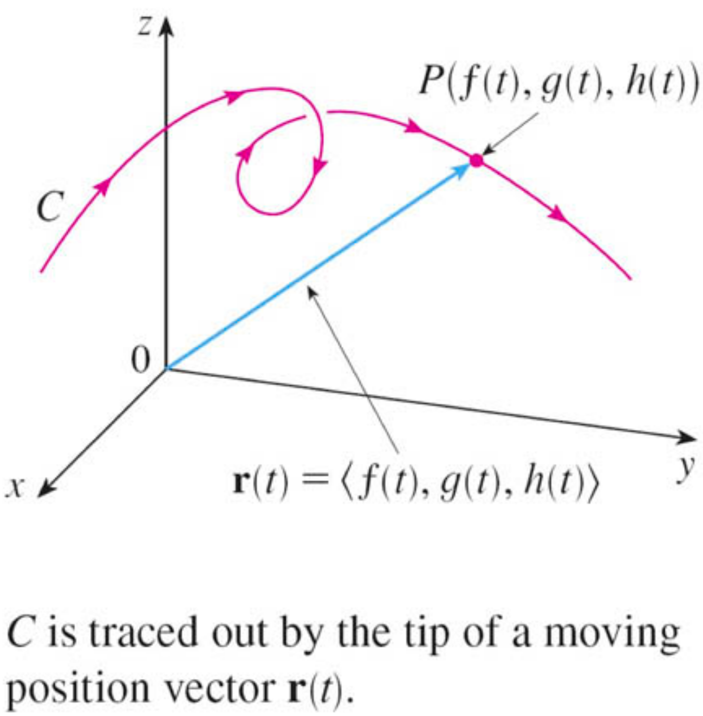
Vector functions [10.1]

Curves in space.

Blessing of the media, Soyuz launch 2013 [NASA]

The **position** of an object in 3-d, as a function of time, $\vec{\mathbf{r}}(t)$ is an example of a **vector function**.



Vector functions and parametric equations

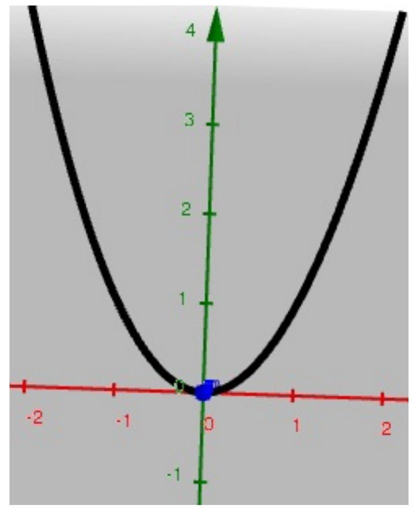
Consider the 2-d vector function:

$$\vec{\mathbf{r}}(t) = \langle t, t^2 \rangle \tag{1}$$

- $\vec{\mathbf{r}}(t) = \langle t, t^2 \rangle$
- $x(t) = t; \quad y(t) = t^2$
- In this case we can write y as a function of $x = t$:

  $$y = x^2$$

  and we know what that looks like...



In GeoGebra: we can make parametric plots, by specifying functions for the x, y and z coordinates of a position vector in terms of a common parameter, for example, the parameter t, like this

```
( [x(t)], [y(t)], [z(t)] )
```

For example

- `(t,t^2,0)` is a parametric plot, where $x(t) = t;\ y(t) = t^2;\ z(t) = 0$, where t runs over all possible values.
- `Curve(t,t^2,0, t,0,10)` is a portion of the parametric plot above, limited to a restricted domain of parameter values: $0 < t < 10.$ >

Try it out!

Parametric plotting of expressions in polar coordinates

What if we'd like to plot a function in polar coordinates? For example:

$$r(t) = 2;\ \theta(t) = t. \tag{2}$$

GeoGebra only allows you to specify functions for the Cartesian coordinates, $x(t)$, $y(t)$, and $z(t)$.

But, you know how to convert polar coordinates to Cartesian coordinates:

$$x = r\cos\theta;\ y = r\sin\theta \tag{3}$$

So, to plot $r(t) = 2;\ \theta(t) = t$, we tell GeoGebra:

```
( 2*cos(t),2*sin(t) )
```

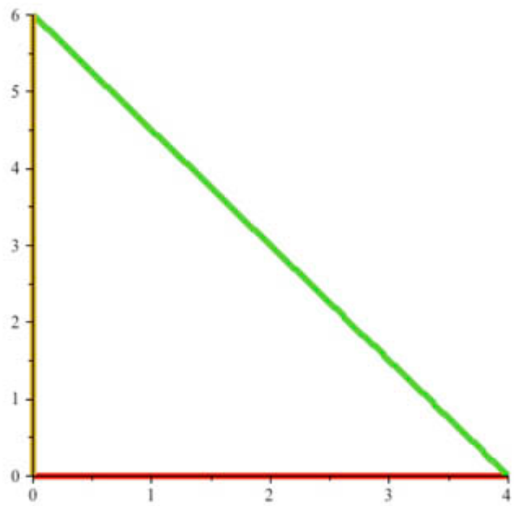
Try it!

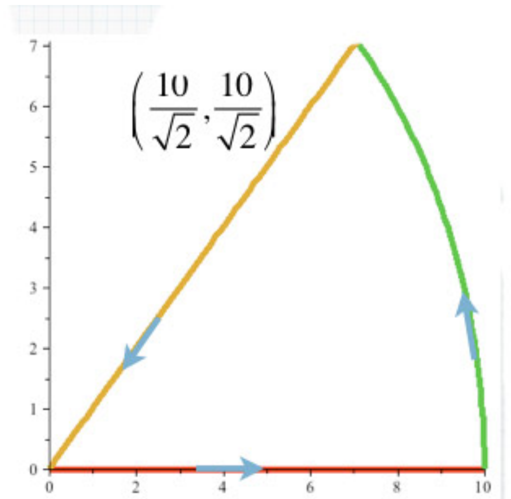
Plotting paths from discrete segments

Using `Curve(...)` you can plot a segment of a curve. Plotting multiple `Curve`s, you can put together the segments to form more complex paths in space.

Plot these paths

Use parametric curve plotting to draw these closed paths in the xy plane ($z = 0$):





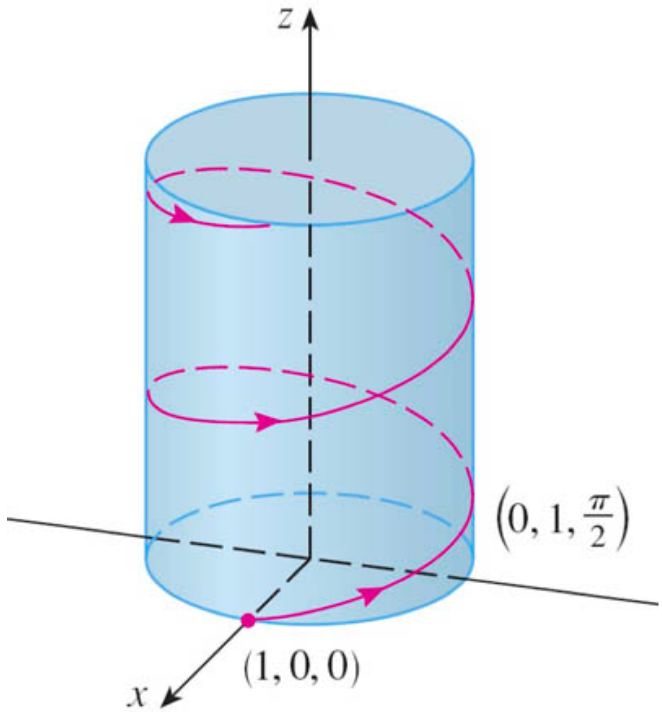
The curved segment above is supposed to be a portion of a circle, with a fixed radius.

Plotting and visualizing curves in 3 dimensions

Consider this curve:

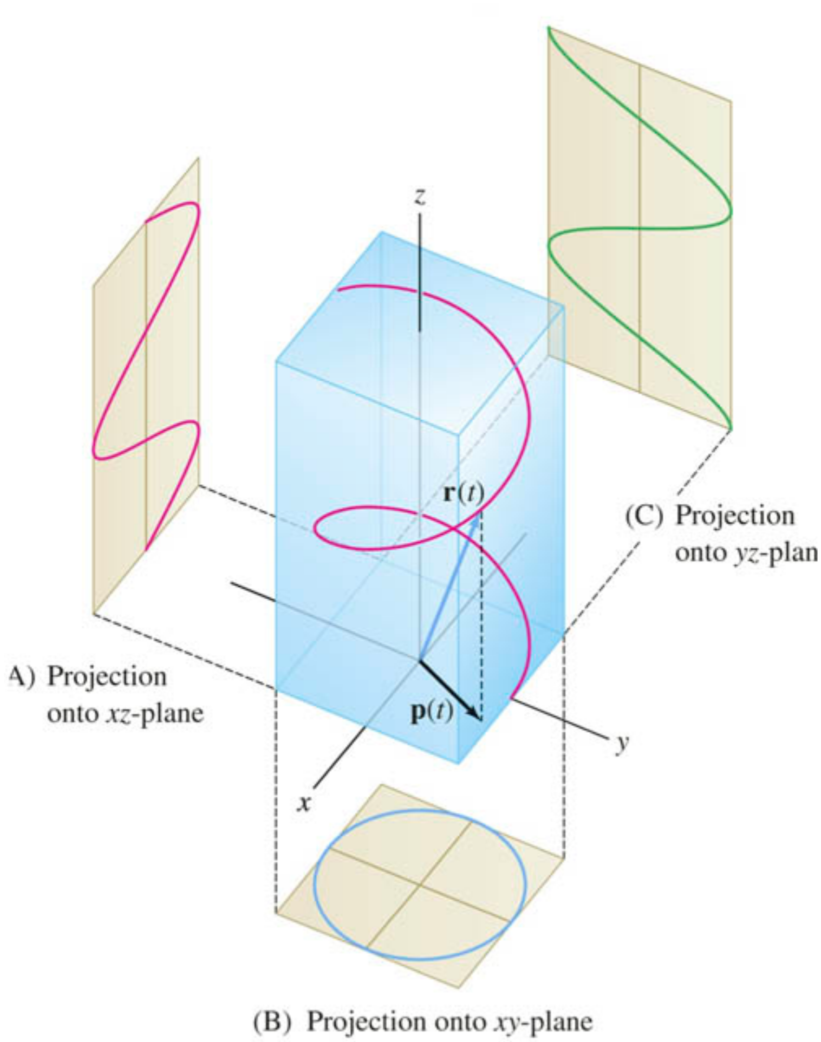
$$\vec{\mathbf{r}}(t) = \langle \cos t, \sin t, t \rangle$$

- $\cos^2 t + \sin^2 t = x^2 + y^2 = 1.$
- In 2-d this is the equation of points on a circle of radius 1.
- So in 3-d, the path traced out by $\vec{\mathbf{r}}(t)$ must lie on the surface of the *cylinder* $x^2 + y^2 = 1$



This process of ignoring one coordinate, and seeing how 2 of the coordinates relate to each other without regard to the other one is "**projection**". Doing this with the other coordinate pairs, we see the familiar-looking relations:

$$x = \cos z; \quad y = \sin z$$



Intersections and projections

Consider

$$\vec{\mathbf{r}}(t) = \langle t, t^2, t^3 \rangle$$

Since $y(t) = t^2$ and $x(t) = t$ we can combine these equations to get

$$y = x^2.$$

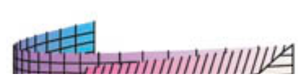
And similarly

$$z = x^3,$$

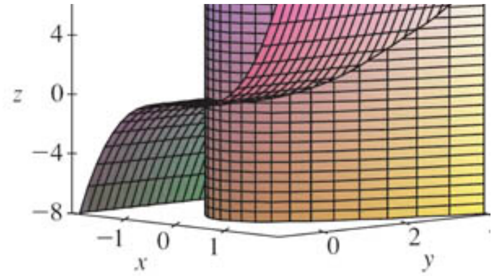
so we could plot the 3-d surfaces

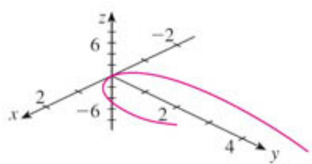
- $y = x^2$ (for any z), and
- $z = x^3$ (for any y)

The points in $\vec{\mathbf{r}}$ need to be on *both* of those surfaces, so we could look for $\vec{\mathbf{r}}$ as the intersection of the surfaces.

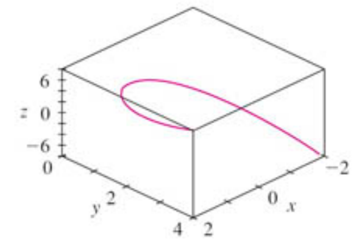


Projections of $\langle t, t^2, t^3 \rangle$

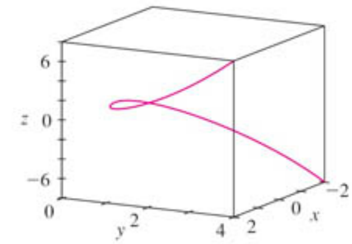




(a)



(b)



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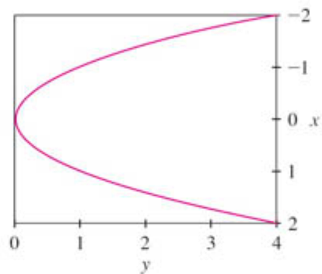

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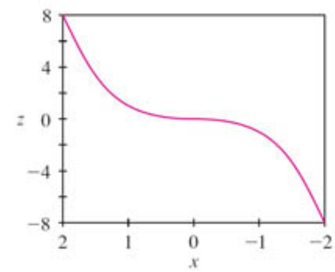

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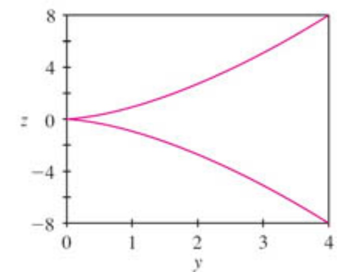

(f)

To Do

- *Handout: Projections*