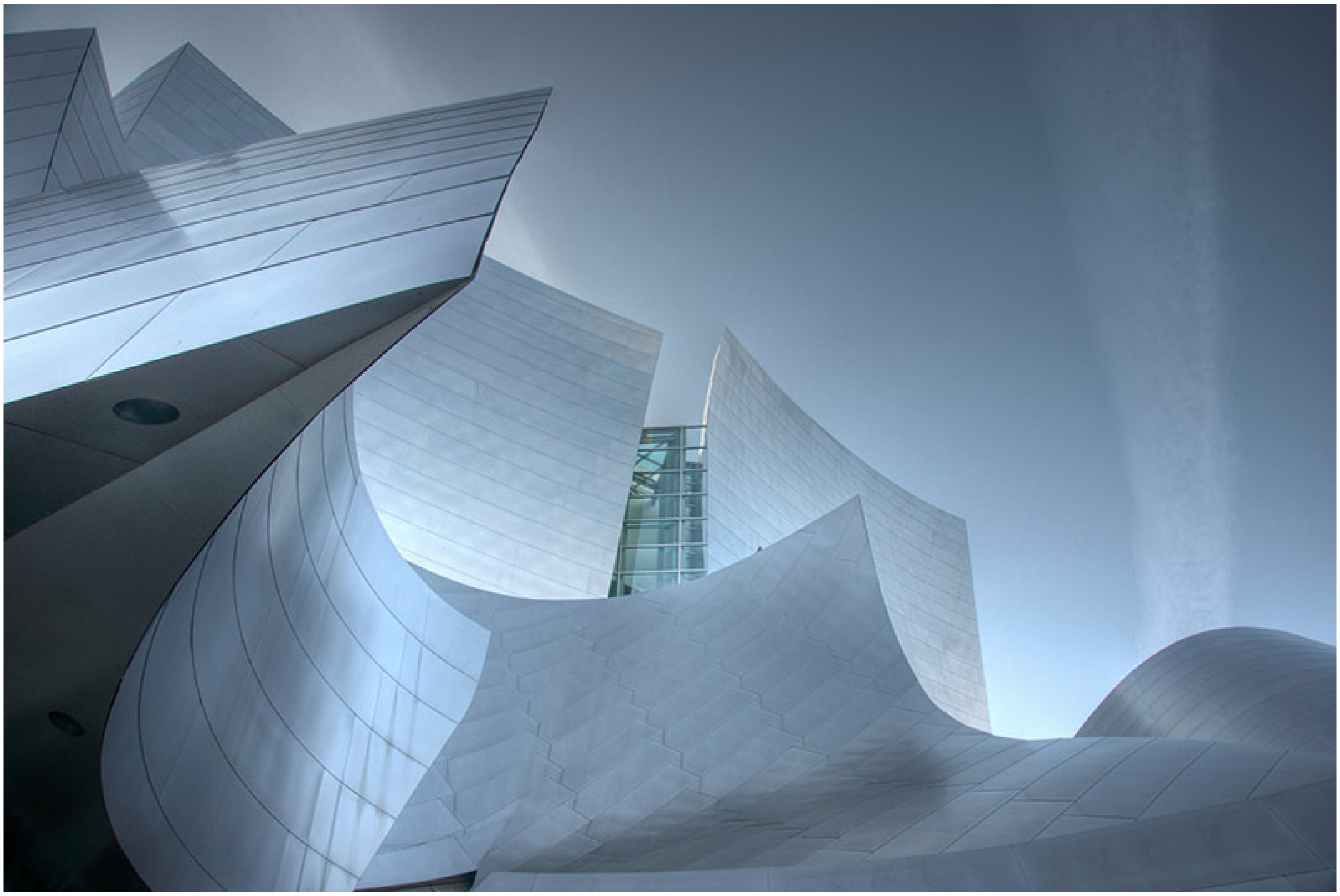
Parametric surfaces [10.5]



Walt Disney concert hall, Los Angeles, architect: Frank Gehry

- Alternate ways to describe a surface,
- In high symmetry situations, a different parameterization may be simpler (than Cartesian

coordinates!)

- Eventually we'd like to answer questions like "how much aluminum will we need for that concert hall roof?".

A vector function $\vec{\mathbf{r}}(t) = \langle f(t), g(t), h(t) \rangle$ of a *single* parameter described a curve in space.

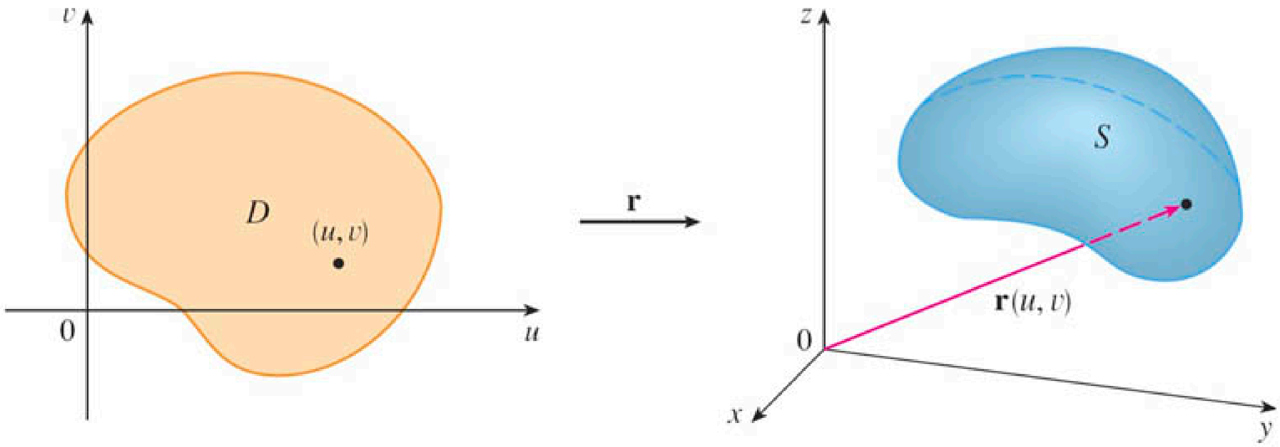
- Suppose that

$$\vec{\mathbf{r}}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$$

is a vector-valued function,

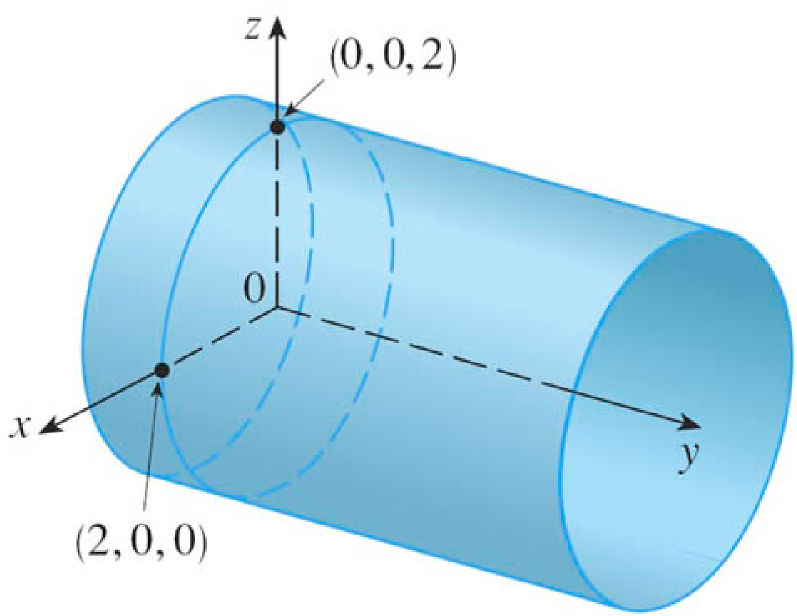
- Defined on a region D of the u- v-plane.

- As (u, v) varies throughout D, then $\vec{\mathbf{r}}$ traces out a **surface** S in 3 dimensions.



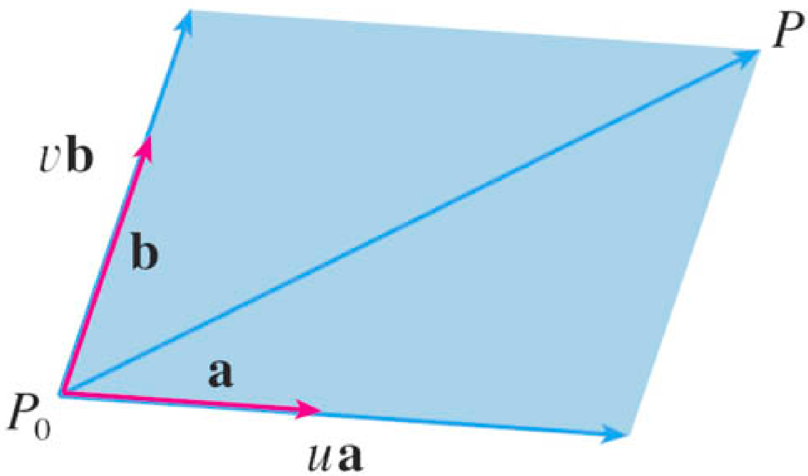
Example: cylinder

$$\vec{\mathbf{r}}(u,v) = \langle 2\cos u, v, 2\sin u\rangle \tag{1}$$



Example: plane

$$\vec{\mathbf{r}}(u,v) = \vec{\mathbf{r}}_0 + u\vec{\mathbf{a}} + v\vec{\mathbf{b}} \tag{2}$$



x and y as 'parameters'

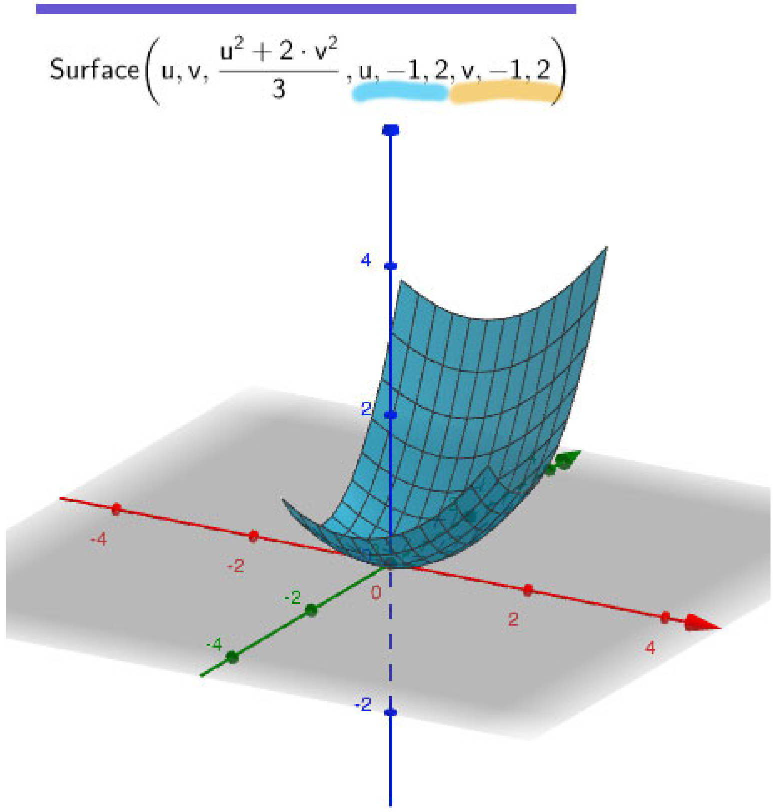
Plot the surface defined by

$$z = \frac{x^2 + 2y^2}{3}. \tag{3}$$

For our two parameters, we could just use $u = x$, $v = y$, and then $z = \frac{u^2+2v^2}{3}$.

Turning this into a vector function of u and v:

$$\vec{\mathbf{r}} = \langle u, v, (u^2 + 2v^2)/3 \rangle \tag{4}$$



Choosing parameters based on symmetries/forms

Consider the points that fulfill:

$$\frac{x^2}{4} + y^2 + \frac{z^2}{4} = 1 \tag{5}$$

You could solve for z:

$$z = \pm\sqrt{4\left(1 - \frac{x^2}{4} - y^2\right)} \tag{6}$$

and plot $\langle x, y, z(x, y)\rangle$.

Or, we re-arrange the equation into

$$x^2 + z^2 = 4(1 - y^2). \tag{7}$$

We recognize the left side as the equation for a circle with a radius $\sqrt{4(1-y^2)}$ that depends on y.

This suggests that we write the *radius* as one of the parameters, $u = 2\sqrt{1-y^2}$ (and inverting: $y = \sqrt{1 - \frac{1}{4}u^2}$), and then we could write x and y in terms of a "polar angle" parameter v sweeping around a circle of radius u like this:

$$\vec{\mathbf{r}}(u, v) = \langle u\cos v, \sqrt{1 - \frac{1}{4}u^2}, u\sin v\rangle \tag{8}$$

Todo

- *Bagels, bagels, bagels*