Functions of several variables [11.1]

Tabular data

Things that depend on more than one variable.

$T_w(T, v)$: A table of wind chill temperature, T_w (in ^{o}C) which depends on the air temperature, T, and the speed of the wind, v.

Wind speed (km/h)

Actual temperature (°C) T \ v	5	10	15	20	25	30	40	50	60	70	80
5	4	3	2	1	1	0	−1	−1	−2	−2	−3
0	−2	−3	−4	−5	−6	−6	−7	−8	−9	−9	−10
−5	−7	−9	−11	−12	−12	−13	−14	−15	−16	−16	−17
−10	−13	−15	−17	−18	−19	−20	−21	−22	−23	−23	−24
−15	−19	−21	−23	−24	−25	−26	−27	−29	−30	−30	−31
−20	−24	−27	−29	−30	−32	−33	−34	−35	−36	−37	−38
−25	−30	−33	−35	−37	−38	−39	−41	−42	−43	−44	−45
−30	−36	−39	−41	−43	−44	−46	−48	−49	−50	−51	−52
−35	−41	−45	−48	−49	−51	−52	−54	−56	−57	−58	−60
−40	−47	−51	−54	−56	−57	−59	−61	−63	−64	−65	−67

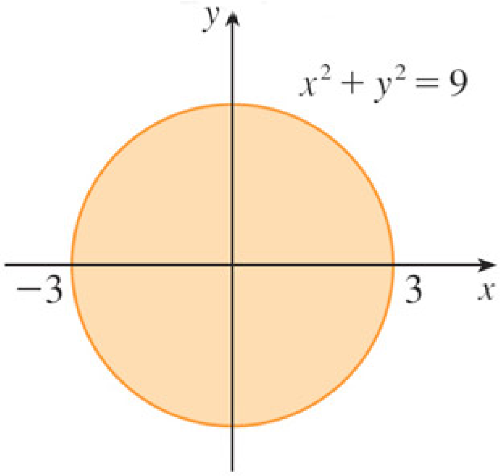
Domain and range

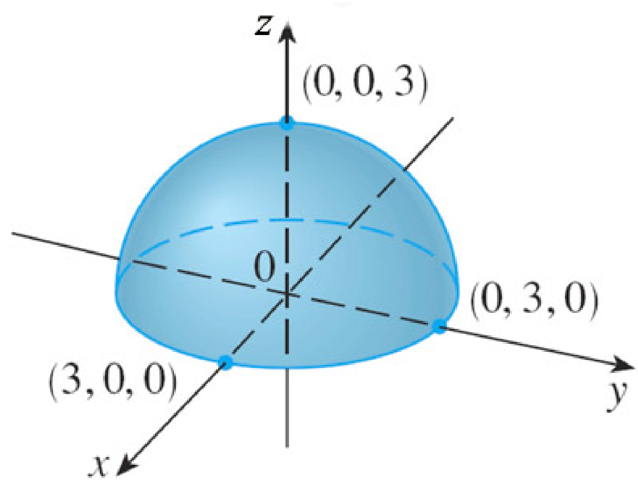
Consider the function

$$g(x, y) = \sqrt{9 - (x^2 + y^2)}. \tag{1}$$

In order that the function evaluate to something non-imaginary, the greatest possible **domain** is

$\{x, y : x^2 + y^2 \leq 9\}$.





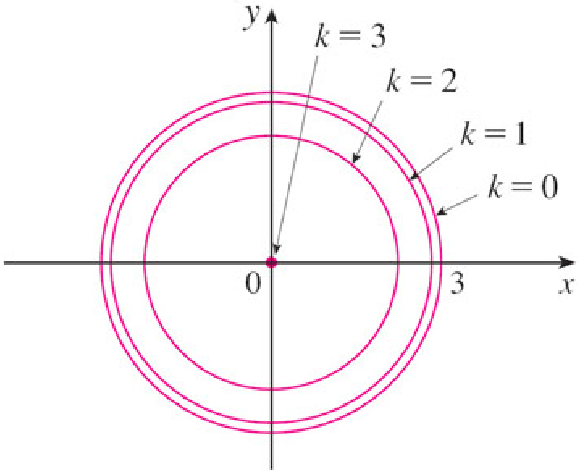
The function can be plotted as a surface, with $z = g(x, y)$, and we see that the corresponding **range** is $0 \leq z \leq 3$.

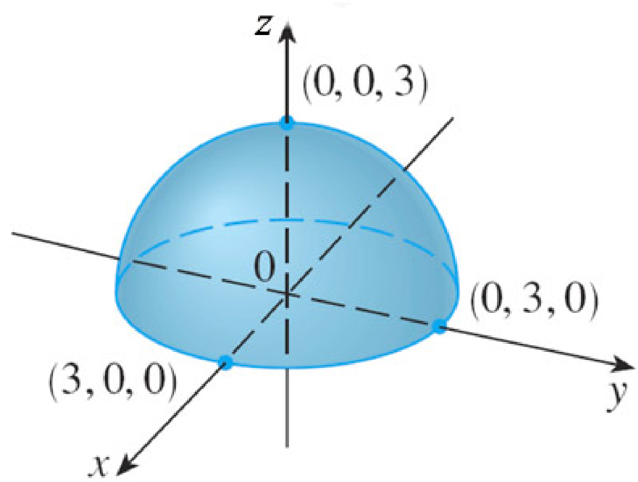
Level curves / contours

One way of visualizing the 3-d surface in 2-d is to plot **level curves**: **Cross-sections** of the surface at a discrete set of **height values** $\{k\}$.

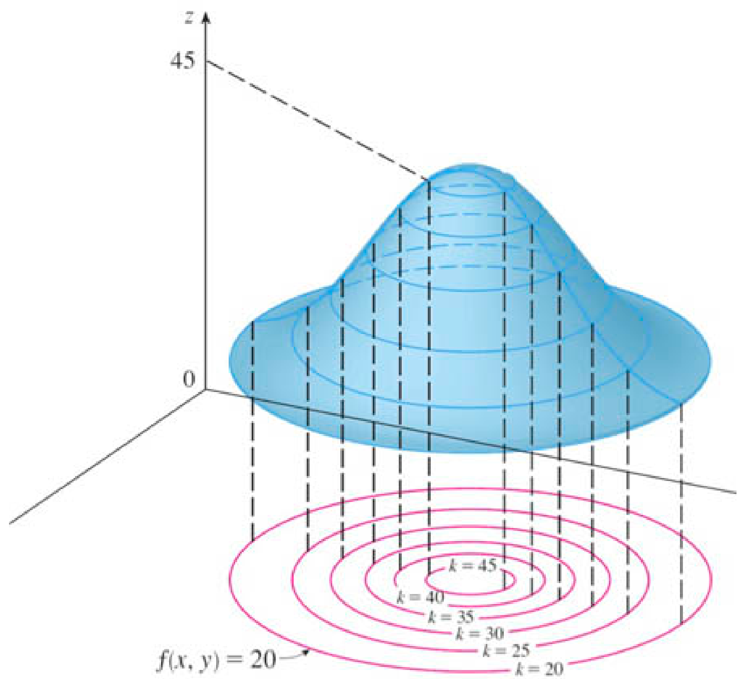
You consider a particular z-height, setting $k = g(x, y)$, and then sketch the resulting curve in the

x-, y-plane.





For a function of 2 variables, $g(x, y)$ we plot level curves $k = g$.



To do

- *Drawing contours*

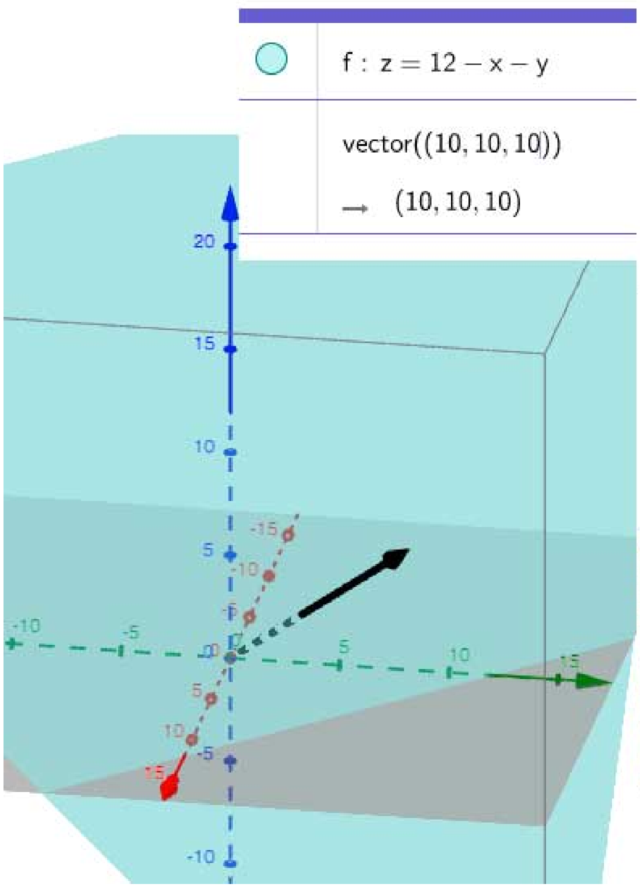
Visualizations

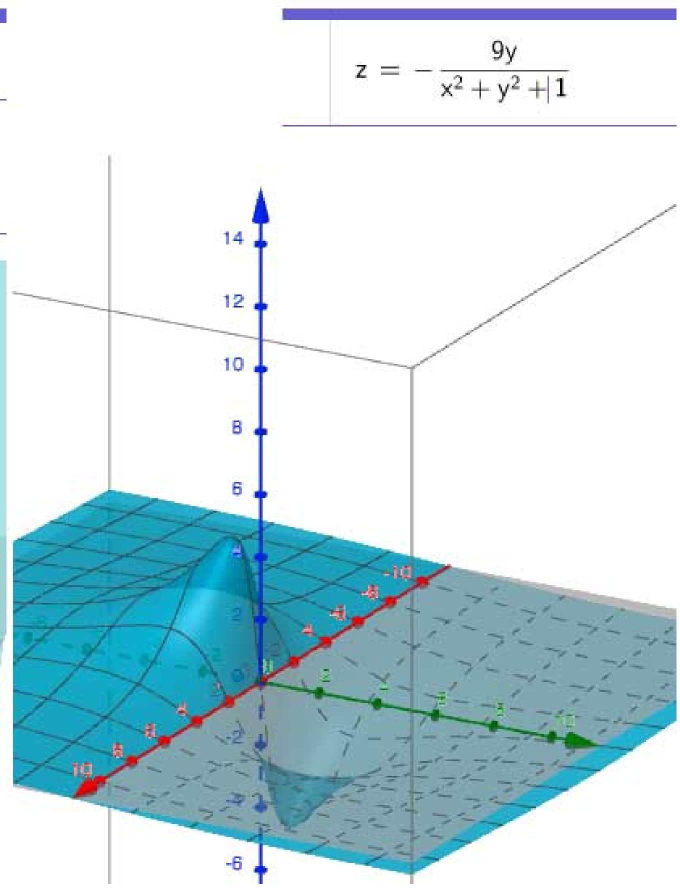
A couple different ways to visualize surfaces in 3-d...

Consider two functions:

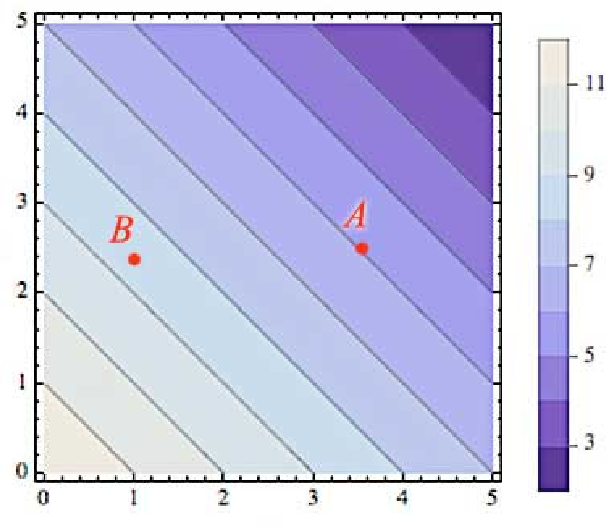
$$f(x, y) = 12 - x - y; \quad g(x, y) = \frac{-9y}{x^2 + y^2 + 1}. \quad (2)$$

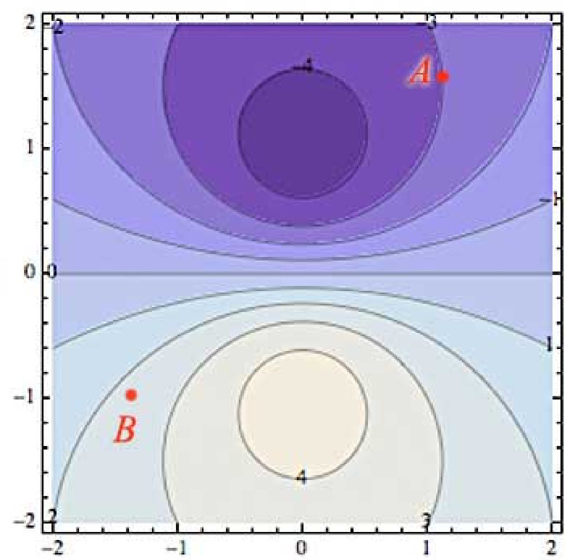
Surface plots





Contour plots





- Sagemath and Mathematica color the **lower** heights **darker** by default.

- *How can you tell* from the contour plot, where the height is changing most rapidly? When the surface is flat?

- *How can you tell* from the contour plot how to move away from point A in such a way that the height will not change?

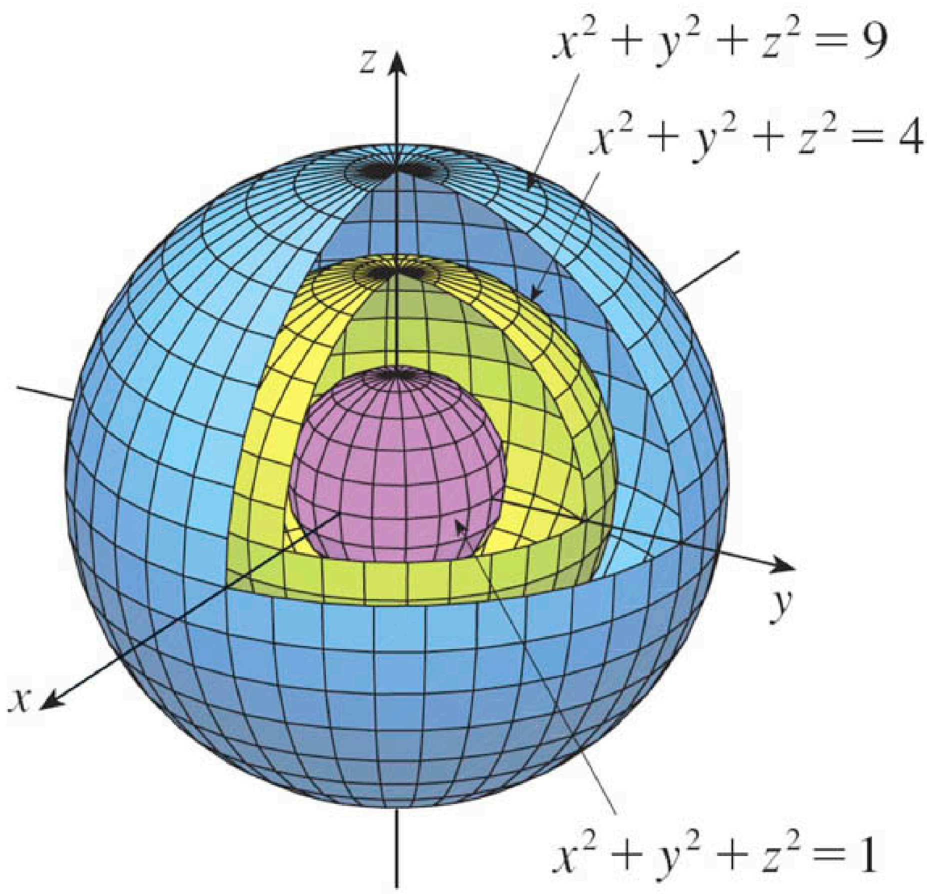
- *How can you estimate* the value of the function at point B?

Todo

- *Using contour plots*

Level surfaces of 3 variables

For a function, $h(x, y, z)$ of 3 variables, the mathematical entities $k = h$ are **level surfaces**.



$x^2 + y^2 + z^2 = 9$
$x^2 + y^2 + z^2 = 4$
$x^2 + y^2 + z^2 = 1$
z
y
x