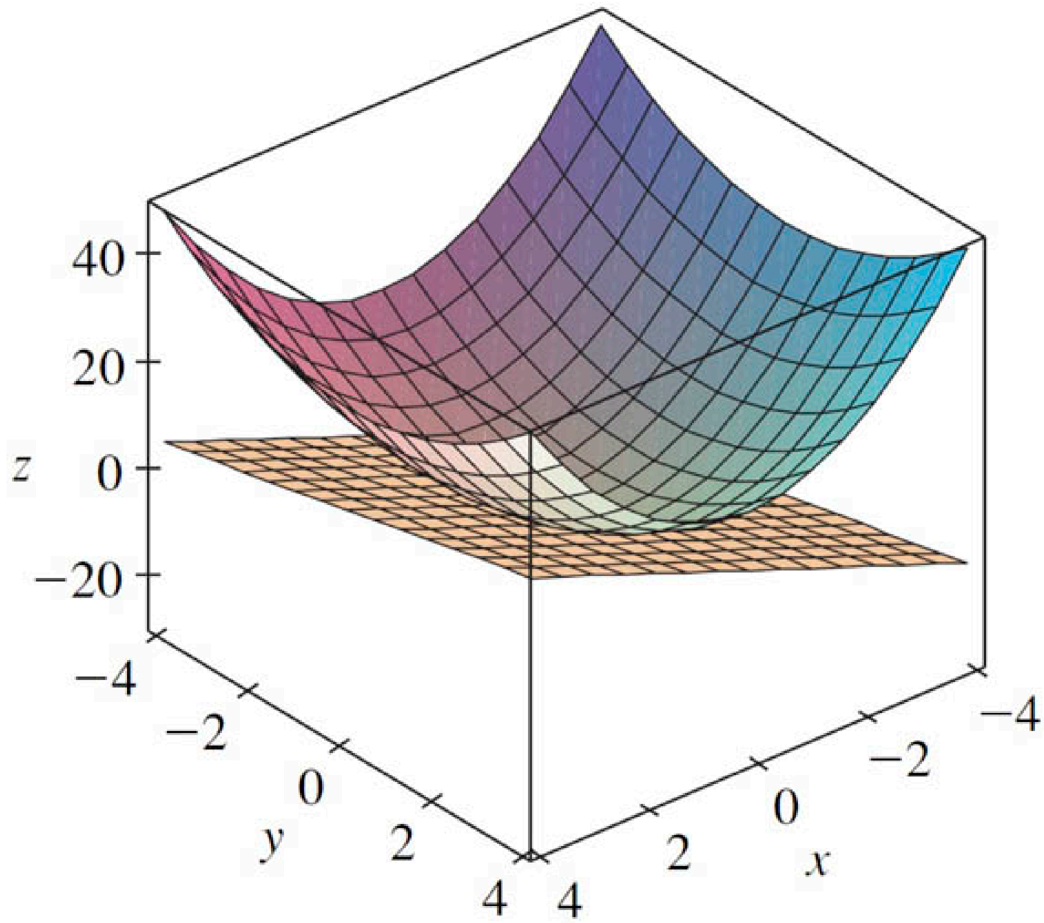
Linear approximations

The board is **tangent** to the bowling ball at the point of contact.

Tangent planes



Suppose f has continuous partial derivaties. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \tag{1}$$

Linear approximation

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b) \tag{2}$$

is called the **linear approximation** or the **tangent plane approximation** of f at (a, b).

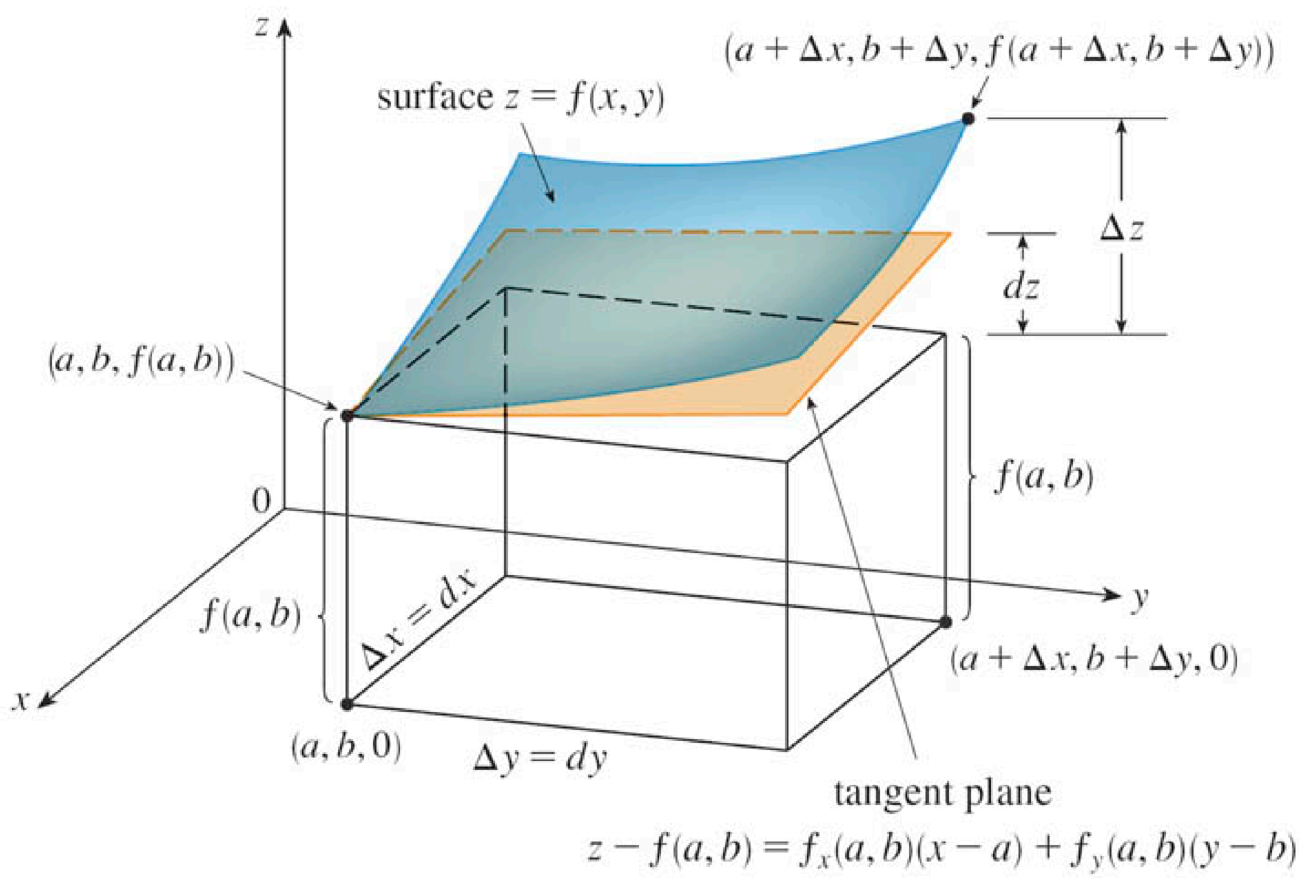
The **differential** of f is

$$dz \equiv f_x(x, y)\, dx + f_y(x, y)\, dy = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy. \tag{3}$$

In the limit of very small 'steps', dx and dy, the linear approximation can be written as

$$f(x, y) = f(a, b) + dz. \tag{4}$$

Geometric view



Hiking view

The hiking approximation

$$\Delta z \approx f_x \Delta x + f_y \Delta y \tag{5}$$

- becomes *exact* as your stepsize becomes small, and
- is apparently equivalent to the linear approximation, and
- only works if there's a unique plane tangent to the surface.

To Do

- *Finish Tabular data*
- *Finish Graphical data*

Image credits

Larry Wilder