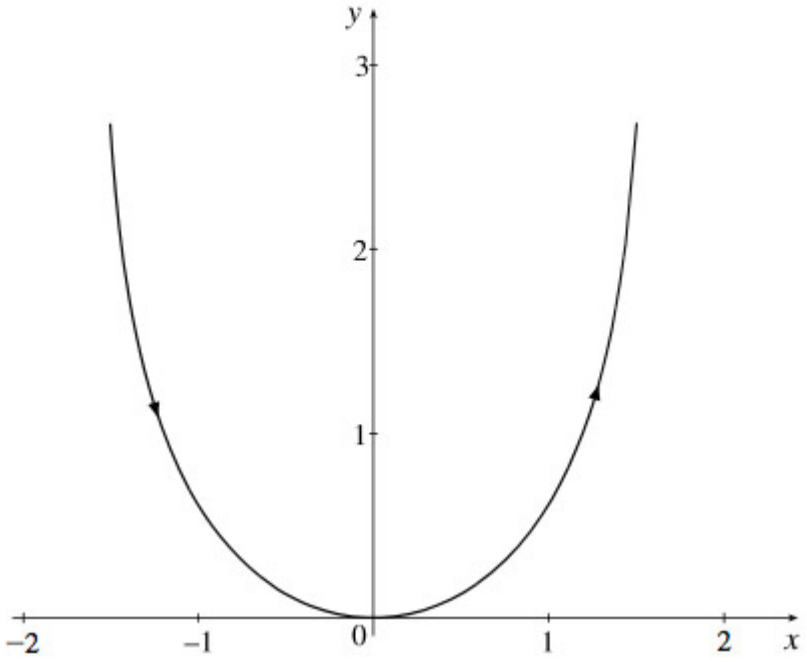
10.2 - the Grim Reaper curve

Consider the curve $\vec{\mathbf{r}}(w) = w\,\hat{\mathbf{i}} - \ln(\cos w)\,\hat{\mathbf{j}}$ on the interval $-\frac{\pi}{2} < w < \frac{\pi}{2}$.

1. Compute the tangent vector $\vec{\mathbf{r}}'(w)$. For each value of w in $\left\{-\frac{\pi}{3}, -\frac{\pi}{4}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right\}$ sketch a tangent vector.



2. For any value of w, what is the length of the tangent vector $\vec{\mathbf{r}}'(w)$? Find an equation for the unit tangent vector $\hat{\mathbf{T}}(w)$.

3. For any value of w, what angle does the unit tangent vector $\hat{\mathbf{T}}(w)$ make with the x-axis?

4. Find a vector $\vec{\mathbf{N}}(w)$ perpendicular to $\hat{\mathbf{T}}(w)$ and pointing away from the curve $\vec{\mathbf{r}}(w)$.