Math 213 - 11.6 - Gradient and Directional Derivative

1. Suppose you are at a the point with coordinates $(0.6, 0.8)$ in a region where the altitude is given by $f(x,y) = \sin(\pi x + 2\pi y)$. In what direction(s) should you go in order to stay at the same elevation? Justify your answer with a brief description of how you solved the problem.

2. When is the directional derivative of a function equal to zero?

3. Suppose that you are given only the following information about a function *f*:

$f(8,5) = 33.1$

$f(8.01,5) = 33.3$

$f(8,5.02) = 33.0$

Estimate

$f_x(8,5)$

$f_y(8,5)$

$\nabla f(8,5)$

$D_{\mathbf{u}} f(8,5)$ where $\mathbf{u} = \frac{1}{5}(3\,\mathbf{i} + 4\,\mathbf{j})$

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4. Some bacteria can sense the "nutritional gradient" and will move in the direction of higher concentration of nutrients. Imagine that such a bacteria is placed at the point P(10,10) in a dish in which the nutrition concentration is given by by $N(x,y)=400-2x^2-y^2$. Sketch the path of the bacteria as it swims towards higher concentrations.

5. The temperature field in the neighborhood of $\left(\frac{\pi}{4},0\right)$ is given by $T(x,y)=\sqrt{2}e^{-y}\cos(x)$.

   Find the path followed by a heat seeking particle originating at $\left(\frac{\pi}{4},0\right)$.