## [12.1] - Double integrals from data

Some of the values of the function $f(x, y)$ are given in the table below:

|  | $\mathbf{x}=\mathbf{2 0}$ | $\mathbf{3 0}$ | $\mathbf{4 0}$ | $\mathbf{5 0}$ | $\mathbf{6 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{y = 8 0}$ | 77 | 78 | 79 | 81 | 82 |
| $\mathbf{8 5}$ | 82 | 84 | 86 | 88 | 90 |
| $\mathbf{9 0}$ | 87 | 90 | 93 | 96 | 100 |
| $\mathbf{9 5}$ | 93 | 96 | 101 | 107 | 114 |
| $\mathbf{1 0 0}$ | 99 | 104 | 110 | 120 | 132 |

Estimate

$$
\begin{equation*}
\int_{y=80}^{100} \int_{x=20}^{60} f(x, y) d x d y \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f\left(x_{i j}^{*}, y_{i j}^{*}\right) \Delta x \Delta y \tag{1}
\end{equation*}
$$

where $\Delta x=20$ and $\Delta y=10$. (That is, the $2 \times 2$ light overlay). Which point to use in each sub-rectangle? Try:

1. midpoints
2. points farthest away from the origin

## [12.1] - Back to the park

The following is a map showing contour lines for a region of Orangerock National Park.


Estimate (numerically) the average elevation by sampling the elevation at the mid-point of each sub-rectangle.

