Lab 07 - The "square-root solid"

This lab has both paper and *Mathematica* pieces. You may hand in one scan of the paper and one Mathematica notebook for both of you who work together.

Consider the volume integral over the solid S given by

$$V = \iiint_S dV = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{z=\sqrt{x^2+y^2}}^{1} dz\, dy\, dx \qquad (1)$$

1. [*Mathematica*] Plot together the surfaces $z = u_1(x, y) = \sqrt{x^2 + y^2}$ and $z = u_2(x, y) = 1$ which are the lower and upper surfaces bounding the volume to be found. Some Mathematica guidance:
   - Use `Plot3D[{function 1 , function 2}, ...]` to plot two functions at the same time.
   - Make both surfaces "see through" with the option `PlotStyle -> Opacity[0.5]`.
   - Make one plot where the x domain is (-1,1) and the y domain is (-1,1).
   - Make a second plot where the x domain is (-1,1) and you restrict y to the domain in the integral, $(-\sqrt{1-x^2},\ +\sqrt{1-x^2})$. What solid is this?

2. Rewrite the volume integral as $V = \iiint_S dz\, dx\, dy$ with appropriate limits.

3. Rewrite the volume integral as $V = \iiint_S dx\, dy\, dz$ with appropriate limits.

4. Using geometry, what should the volume of this solid be? Justify your answer.

5. [*Mathematica*] Use Mathematica to evaluate all three of the integrals you've written out. They should agree! Guidance:
   - Pull up the Writing Assistant from the Palettes menu.
   - Don't use `Integrate[ ]`. Instead, from the typesetting pallette (in a Wolfram Language cell), insert the definite integral object and fill in the limits and integration variable, so that the triple integral *visually* matches the one you're trying to find.
   - Work from the outside of each triple integral to the innermost integral. You'll insert the definite integral object 3 times for each triple integral.

6. Referring back to the integral at the beginning of this assignment: Evaluate the dz integral, then convert the remaining double-integral to polar coordinates. Write down (below) your double integral in polar coordinates and then evaluate the result [*Mathematica*] to show (hopefully!) that this also gives the same answer.