Lab 04 Vector-Valued Functions

[Your name(s) here!]

Examples to steal from

Here are examples of some of the kinds of things you'll do in this lab.

Plotting a parametric vector function:

This is really a vector function $\vec{r}(t)$ where the vector is a position vector. It's got 3 functions for x(t), y(t), and z(t) in its declaration, But I'm going to risk a little clarity by writing it without bothering to put the vector over top...

```
r[t_] := {t, t Sin[t], t Cos[t]}
ParametricPlot3D[r[t], {t, 0, 8 π}, PlotStyle → Blue]
```

Displaying a point on the curve, together with the derivative vector at that point.

```
Show[
 ParametricPlot3D[r[t], {t, 0, 8 π}, PlotStyle → Blue],
 Graphics3D[   {
   (* Big red dot at r[t=3.5 π] *)
   Red, PointSize[.04], Point[r[3.5 π]],
   (* Arrow[ ] requires coordinates of tail, then head *)
   Arrow[ {  r[3.5 π], r[3.5 π] + r'[3.5 π] } ]
  } ]
]
```

Animating the "particle"

```
Animate [
 Show[
  ParametricPlot3D[r[k], {k, 0, 8 π}, PlotStyle → Blue],
  Graphics3D[   {
     (* Big red dot at r[t=3.5 π] *)
     Red, PointSize[.04], Point[r[t]],
     (* Arrows require coordinates of tail, then head *)
     Arrow[ {  r[t], r[t] + r'[t] } ]
    } ]
 ],
 {t, 0, 8 π}
]
```

Speed: Speed is the magnitude (or "norm") of the derivative vector: $|\vec{r}\,'(t)|$. Here's how you can graph the (scalar) speed as a function of time:

```
Plot[   Norm[r'[t]]   , {t, 0, 8 π}]
```

Exercise 1

```
N[{E, e}]
```

4 pts Consider the two arcs $\boldsymbol{a}(t) = \langle e^t \sin t,\ e^t \cos t,\ e^t \rangle$, $0 \le t \le 2\pi$ and $\boldsymbol{b}(t) = \langle t \sin(\ln t),\ t \cos(\ln t),\ t \rangle$, $1 \le t \le e^{2\pi}$. [In Mathematica, type E (not e) to get the base of the natural logarithm.] Plot both space curves and show that they trace out identical curves. You can display two plots sides by enclosing them in list brackets, like { ParametricPlot3d[...], ParametricPlot3d[...] }.

Exercise 2

6 pts In order to examine how a particle moves according to each function, create an animation of each like this: First create a display of the arc and a large dot located at $\boldsymbol{r}(t)$. Then "`Manipulate[..]`" the plot to move the dot as t is varied with a slider. Finally substitute `Animate` for `Manipulate`. This will change t at a constant rate. (See if you can get the derivative vector to show as well).

Comment on differences that you observe between the two arc functions.

Exercise 3

4 pts Calculate $\boldsymbol{a}'(t)$, $|\boldsymbol{a}'(t)|$, $\boldsymbol{b}'(t)$, and $|\boldsymbol{b}'(t)|$. Graph the speeds. Describe how these results support (or challenge) the observations you made above.

Exercise 4

10 pts Use Mathematica to calculate the arc length separately for each vector function. Use the formulae we developed in class, which involves integrating the “speed”...

$$L = \int_a^b \left| \vec{r}\,'(t) \right| dt$$

Calculate the arc-length using both ***a***(*t*) and ***b***(*t*)--they should agree if the space-curves are identical.

And then use Mathematica’s built-in `ArcLength[...]` function using both ***a***(*t*) and ***b***(*t*) and see if that function gives the same result.