## Lab05 - Partial derivatives - part 2/2

## [Your name(s) here...]

We've been working with the function defined (in function syntax) below, and evaluating partial derivatives at $(x, y)=(\pi / 2,1)$.

$$
z\left[x_{-}, y_{-}\right]:=y \cos \left[x y^{3}\right]
$$

Your next step will be to make a contour plot of this function, and then estimate some of the second derivatives of the function. Here, for you reference, is the picture of the heat-seeking kitten contour plot, which will serve as an example of some of the things you can do to complete this lab:

```
\(\ln [1]:=\mathbf{m y T}=\mathbf{x}^{\wedge} \mathbf{2 - 2} \mathbf{x} \mathbf{y}\);
    myCP = ContourPlot [myT, \(\{x, 0,1\},\{y, 0,1\}\), Contours \(\rightarrow 40]\);
    myPoint|=
        Graphics [\{PointSize[Large], Black, Point[\{1/2, 1/3\}]\}];
    myBound = Plot \([x,\{x, 0,1\}]\);
    Thickness [0.05]
    myQuad = ParametricPlot \(\left[\left\{t, t^{\wedge} 2\right\},\{t, 0,1\}\right]\);
    Show [myCP, myPoint, myBound, myQuad]
Out[[]]= Thickness[0.05]
```



## Problem 54 pts

Using your single variable plots (Problem 2, from part 1 of this lab), estimate visually whether these second derivatives (related to the curvature of your graph) are positive, negative, or zero:
$f_{\mathrm{xx}}(\pi / 2,1) ?$
$f_{x x}(1,1) ?$
$f_{y y}(\pi / 2,1) ?$
$f_{y y}(\pi / 2,1 / 2) ?$

## Problem 64 pts

Make a contour plot of $z(x, y)$. See the kitten example for how to place a dot on your contour plot. Include a dot at the coordinate $(x, y)=(\pi / 2,1)$.

Using your contour plot, estimate visually whether these second derivatives are positive, negative, or zero:
$f_{\mathrm{xy}}(\pi / 2,1) ?$
$f_{y x}(\pi / 2,1) ?$

## Problem 74 pts

To take a 2nd derivative in Mathematica you can "chain" D[...] functions together. For example, taking the partial derivative of a function with respect to $x$ or $y$. And then taking the derivative of *that* with respect to $x$ or $y$.

Use Mathematica to calculate the following second derivatives exactly and compare to your visual estimates above.
$f_{\mathrm{xx}}(\pi / 2,1) ?$
$f_{y y}(\pi / 2,1) ?$
$f_{\mathrm{xy}}(\pi / 2,1) ?$
$f_{y x}(\pi / 2,1) ?$

