Lab 09 - Vector Line Integrals

[Names]

Writing a force field as $\vec{F} = P\,\hat{i} + Q\,\hat{j}$, the vector line integral over some path C can be written in at least two ways:

$$\int_C \vec{F} \cdot d\vec{r} = \int_C P\,dx + \int_C Q\,dy$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F}(x(t), y(t)) \cdot \vec{r}\,'\,dt$$

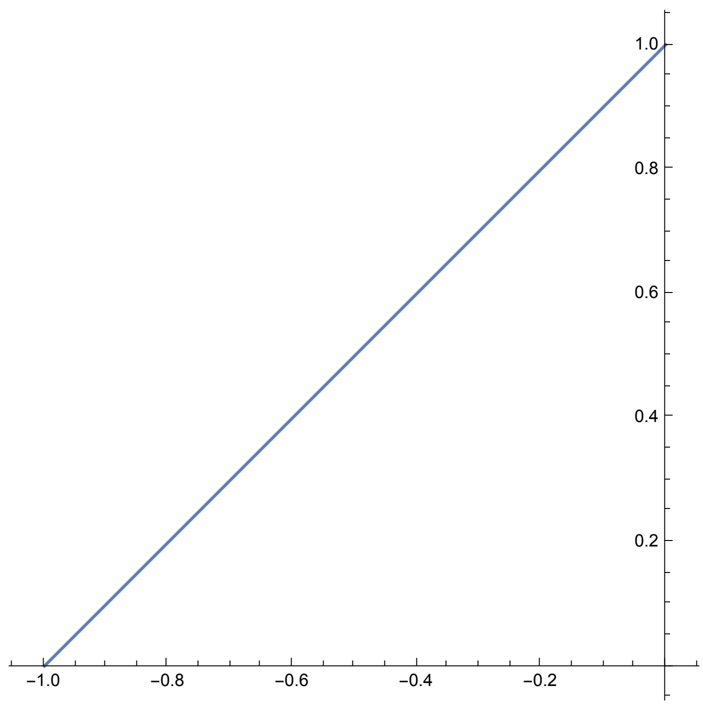
where $\vec{r}\,' = \left\langle \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t} \right\rangle$. Use whichever of these expressions is most useful in evaluating the following line integrals.

Write down the answers you get here in your *Mathematica* notebook, but attach paper showing your work (and labelled according to question number) to a printout of the notebook.

1.) Consider the force field $\vec{F}_1 = \vec{r} - 2\,\hat{i} = x\,\hat{i} + y\,\hat{j} - 2\,\hat{i} = (x-2)\,\hat{i} + y\,\hat{j}$. Evaluate the vector line integral of this force field along the path C_1: the straight line from (-1,0) to (0,1).

[**Hint:** It's probably easiest to do this using the first relation above.]

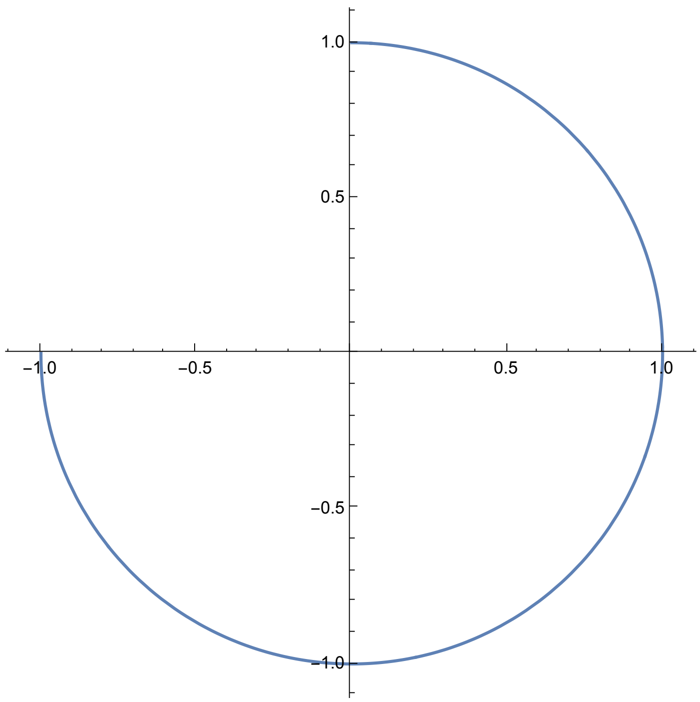
```
C1 = ParametricPlot[{x, x + 1}, {x, -1, 0}]
```



2.) Evaluate the vector line integral of the same force field, $\vec{F}_1$, but this time along the path C_2: the portion of a radius one circle, traversed in counterclockwise direction from (-1,0) to (0,1).

[**Hint:** It's probably easiest to do this one using the *second* relation above, and using the parameter θ instead of t. You can use *Mathematica* to evaluate any trig-function-containing integrals you come up with.]

```
C2 = ParametricPlot[{Cos[θ], Sin[θ]}, {θ, -π, +π / 2}]
```



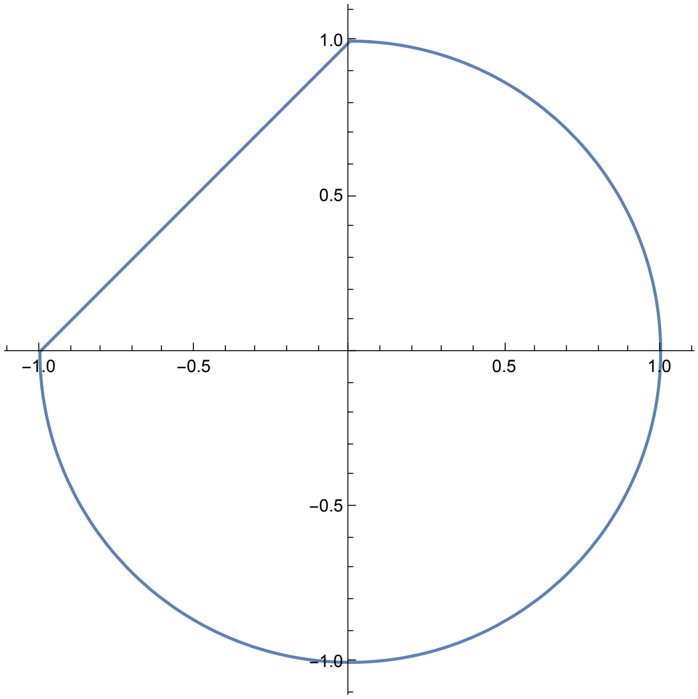
3.) Plot the vector field $\vec{F}_1$ (use red arrows, see Lab 08) together with the integration paths $C_1$and C_2. Do the *signs* of your vector line integrals match your visual estimation of their signs?

4.) Could $\vec{F}_1$be the gradient of some potential function $f_1(x, y)$? Carry out the curl test to decide. (Show the results of your test). Investigate how to use *Mathematica*'s `Curl[....]` function to confirm. Would you have guessed this result (well, whether it's positive, zero or negative) by looking at the graph of the vector field arrows?

If the field is "irrotational", find a potential function $f_1$such that $\vec{F}_1 = \vec{\nabla} f_1$ and make a plot which includes the integration paths, the vector field of arrows *and* a `ContourPlot[...]` of f_1-- all together in the same figure.

5.) If the potential function exists, calculate the surface height difference $f_1(0, 1) - f_1(-1, 0)$.

```
Show[c1, c2]
```



6-10.) Repeat the sections 1-5.) with the same curves C_1 and C_2, but this time with the vector field $\vec{F}_2 = \left(-y - \frac{1}{2}\right)\hat{i} + x^3\,\hat{j}$. (And try to find a potential called f_2 if possible.)

11.) Back to the first vector field, $\vec{F}_1$: Carry out a vector line integration of this vector field from the origin to a point with coordinates (x_0, y_0) along a path with two segments:

First go along the x axis from (0,0) to $(x_0, 0)$.

Next go from $(x_0,0)$ to (x_0, y_0). This second path is parallel to the y axis.

Now, replace $x_0 \to x$ and $y_0 \to y$. Comment on the result.